



**Elektriseadmete raalprojekteerimine kursuse  
metoodiline juhend: transformaator**

**Metoodiline juhend üliõpilastele ja juhendajatele**

**Studying guide in course on Numerical modelling and  
design of electrical devices: transformers**

**Guide for students and supervisors**

**Project 1.0101-0278**

**Application of interdisciplinary and international team  
and project based learning in Master Studies**

**IN 557**

# Studying guide

## Objective

A construction and some design guidelines of the low-voltage low-power transformers are described in this paper. The guidelines allow student to proceed through the design process, modelling and analysis of the small-size transformers for adapted power supplies. As a result a technical specification can be composed in order to build a prototype model.

This material could be efficiently used when studying or teaching transformers and could be a complementary material for some classical textbooks describing transformer construction and design. The purpose of this material is to achieve a complete design experience, where much learning takes place, so that the student gets a good “hands-on” experience of exhaustively analysing, designing and finally prototyping the device. The material avoids establishing a design model that could be directly run in computer software in order to facilitate theoretical part of the work. The material rather tries to give some calculation examples and analysis that could be used in the design model that is actually engineered by a student. Guiding questions “Q” should encourage the development of a creative thinking and an intelligence of analysis.

This design guide for students and supervisors are used during individual preparation period and it is continued in the team-work session to study the design of transformers focusing on energy conversion efficiency, cost effectiveness and cost analysis.

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# 1 Introduction

A learning process that bases on the individual efforts and develops the theoretical understanding and the practical experience has undoubtedly a great input to the education of electrical engineers. This particular course bases on the numeric modelling, design and prototyping of electrical devices – in particular small power supply transformers. The objective in this course is to achieve a *complete design experience* including the design, the actual construction and the testing of an electromagnetic device such as a transformer [18] [24][25]. It is important to utilize the advantages of computer aided design and manufacturing (CAD/ CAM) in order to make a device that meets all the performance requirements of the device at minimum cost. A good design is an optimal utilization of geometry, properties of media and performance requirements. Note that the design *goal is to exceed*, not to just meet, the performance and reliability requirements. This comes from the philosophy that given fixed resources the engineer's task is to get the most from these resources.

The design guidelines are focused on low and fixed frequency domain. In the longer perspective the guideline material should be extended in order to design or to evaluate any kind of *electromagnetic energy converter* – a transformer that operation mode is

- Conventional
- Switching
- Resonance

and the common purpose is

- Power transfer
- Power conditioning
- Galvanic separation

As a matter of fact the guideline material in the present size focuses to the low frequency continuous mode single phase transformer in order to establish a good physical understanding, which is essential in the computer aided design process [5][6][8][9][12][13][15][16][20][21][23]. Therefore it could be frustrating that the problems in electromagnetism are handled almost like static by neglecting the influence of the time dependent fields. Anyway the switched mode power supply (SMPS) will be a continuation to the present material [22].

## 1.1 Design goals

The *performance requirement* for the power transformer or the transformer functionality is obtained by the electrical design factors, which are related to the supply and to the load such as power, voltage, frequency and number of phases and the voltage regulation. The device of a certain power supposes to fulfil certain requirements to insulation in order to withstand the temperature and avoid flashovers. Thermal requirements and cooling conditions are stated in the specification. Additionally the mechanical or electromagnetic conditions such as audible noise or shielding, respectively, can be specified. Mechanical strength requirement, vibration, resonant modes, etc are usually not prioritized as a design goal.

## 1.2 Design constrains

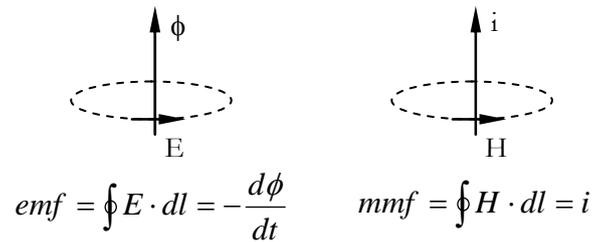
The transformer principle of operation bases on Faraday's law of induction. The challenge of designing a transformer is to solve a coupled problem of multiphysics. This incorporates electromagnetism, mechanics, heat transfer and the coupling between the different fields that is all in accordance with energy conversion principle. *Theoretical design factors* are used to quantify the physical parameters of the energy conversion process [19]. They can be broken into five areas: electrical, dielectric, magnetic, mechanical and thermal. These can be also seen in turn as different circuits with flows and flow densities, which are coupled into geometry. The flow densities such as the electrical current, the magnetic flux and the loss power density can be considering as the loadings that establish the relations between the size, the material properties and the performance of the energy converter. In the case of a transformer one of the design constrains is to model and specify properly its behaviour in the circuit application (including the effects of multiphysics).

## 1.3 Design optimization

In order to optimize a transformer it is necessary to find an appropriate geometric distribution of materials and sources that fulfil the performance requirements – a *device model*. The optimal design of an electric device is to choose a set of dimensions for the device (design variables), so that the designed device will satisfy the design criteria (constraints) and minimize (or maximize) some additional criterion (objective function). The objective functions, which could be chosen such as minimal losses, minimal cost (and temperature) [3], minimal weight etc, are optimized [4][7].

## 2 Construction of a transformer

A transformer has at least a pair of coils *coupled magnetically* together. As a consequence some of the magnetic flux produced by current in one of the coils links the turns of the other(s), and vice versa. Magnetic flux and electric current ‘indicate’ the flow and the presence of magnetic and electrical circuit(s). The fundamental purpose of any magnetic core or a circuit is to provide an easy path for flux in order to facilitate flux linkage or magnetic coupling between two or more magnetic elements. In an electromagnetic energy converter the magnetic elements are primary and the secondary windings. The electromagnetic coupling between the magnetic and electric circuits obeys Ampere’s and Faraday’s law (Figure 2.1).



$$emf = \oint E \cdot dl = -\frac{d\phi}{dt}$$

$$mmf = \oint H \cdot dl = i$$

Figure 2.1 Ampere’s and Faraday’s law describing the static and the dynamic connection between the electrical and the magnetical quantities. The reason for an electrical current is the electromotive force (emf) such as for the magnetic flux is the magnetomotive force (mmf).

Transformer can be seen as an electromagnetic inductor that is loaded with a secondary winding. As a matter of fact a transformer that instantaneously converts energy between the primary and the secondary winding have to have high magnetic permeability of the magnetic circuit as the *energy storage* in the transformer core is undesired parasitic effect. On contrary, a transformer operating in switching mode needs to have intermediate energy storage that means a low net magnetic permeability of the circuit.

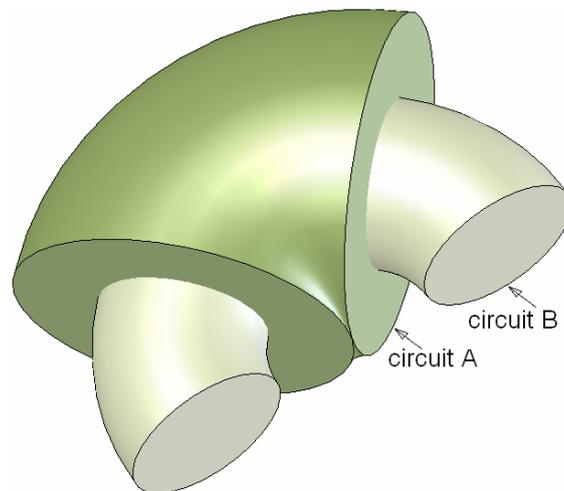


Figure 2.2 Electromagnetic circuit with the shortest ‘contour’ length. The origin of shell and core type.

One possible, a geometrically ideal coupling of electric and magnetic circuit is shown in Figure 2.2. It would not be complicated to imagine that ‘circuit A’ could be the electric circuit and ‘circuit B’ is the magnetic circuit, or vice versa. From functionality point of view the electric circuit may consists as many separate windings as needed. The windings are wound close to each other in order to

reduce leakage flux and to reduce a voltage regulation. From manufacturability point of view, a winding is composed from a number of electrically insulated wire strands so that the necessary ampere turns or voltage per turn can be achieved. Similarly, the magnetic circuit could be made from a magnetic wire. For the medium and high power, low frequency transformer it is important that the magnetic core has low permeability, low magnetization losses and the core could be easily assembled. This condition is fulfilled by using a stack of laminated electromagnetic steel sheets.

## 2.1 Transformers with a laminated core

Two *general configurations* of iron and copper in a transformer are shell or core type of transformer. Figure 2.3 shows the cross-section of a transformer that is in magnetic flux plane and perpendicular to the electric current. Figure 2.4 shows on contrary the cross-section of a transformer that is in electric current plane and perpendicular to the magnetic flux. Further investigation of the different core types is described in chapter **Error! Reference source not found.** Most small power transformers are shell-type, in which the iron surrounds the copper. The magnetic circuit is divided into two return paths on opposite sides of the core, as is done with the E and I laminations, as shown. Another configuration, the core type, in which the copper surrounds the iron, is common with larger high-voltage transformers, as well as with small ferrite-core toroidal transformers. Note that both primary and secondary windings are placed on each leg. The laminations are L-shaped to make a core of uniform cross-section with a central rectangular window. Core transformers are expectedly easier to insulate and to cool. If the primary winding were placed on one leg and the secondary on the other, as in the diagrams of transformers in texts, leakage flux would be excessive, and the transformer would have poor regulation (variation of voltage with load). This kind of transformer analysis is described in chapter 4. There are other special types of construction such as distributed core, etc, but all can be classified as shell or core.

The important feature of the transformer core, which is made as a stack of laminations, is a *high permeability*. This can be achieved, especially in grain oriented electromagnetic steels, where the materials have a high permeability in the rolling direction and not as good any other direction. Grain-oriented steel is inherently suitable for wound-core construction (right column in Figure 2.3 and Figure 2.4). The simplest way to build a wound core is to wind a continuous strip of steel on a rectangular form. When the desired thickness has been built up the core is split into two halves for a single magnetic flux loop. If the planar laminations are the same or the constant width of the steel strip in the wound core, it is obvious that the magnetic circuit will have a rectangular cross-section. However, if the narrower laminations are used for the outer layers, then it is possible to get more-or-less round cross-section of the magnetic core.

Another important feature of the laminated transformer core is the *magnetization losses*. If the lamination thickness is reduced by factor  $k$ , then the eddy current loss is reduced by a factor  $k^2$ . The eddy current losses are proportional to electric resistivity of the material. Hysteresis losses are usually independent of material impurity, which results at the same time higher resistivity and hysteresis losses. The fringing fields in the lamination joins and stray flux in the perpendicular direction of the lamination plate are significant source of losses and the local heating.

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- Q.1 What kind of shape should have discrete laminations in order to form a core and which cases the advantage can be taken of the grain oriented steel?
- Q.2 What kind of shape and proportions suppose to have a scrapless lamination (the laminations stamp from a rectangle of electromagnetic steel sheet has no waste left over)?
- Q.3 How should look like the corresponding shell and core type transformer (2D/3D) for a three-phase transformer?
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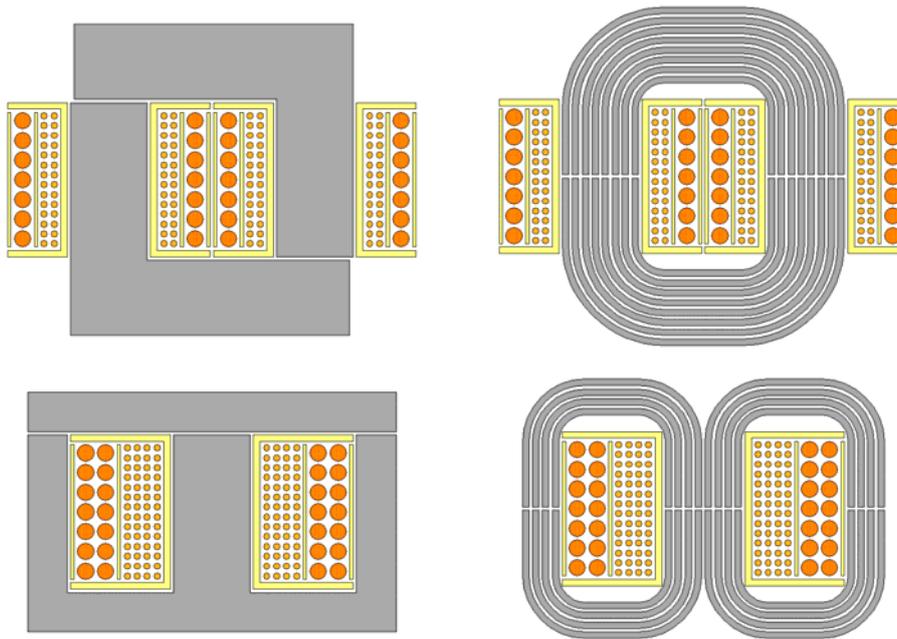


Figure 2.3 The cross-section of a transformer on the plane of magnetic flux flow. The upper row indicates a core-type transformer and the lower row a shell-type of transformer. The transformers with planar laminations are shown in the left column and the wound-core transformers in the right column. If the same electric conductor cross-section is considered then the core-type of transformer are expectedly longer compared to shell-type of transformers, which is due to additional insulation material.

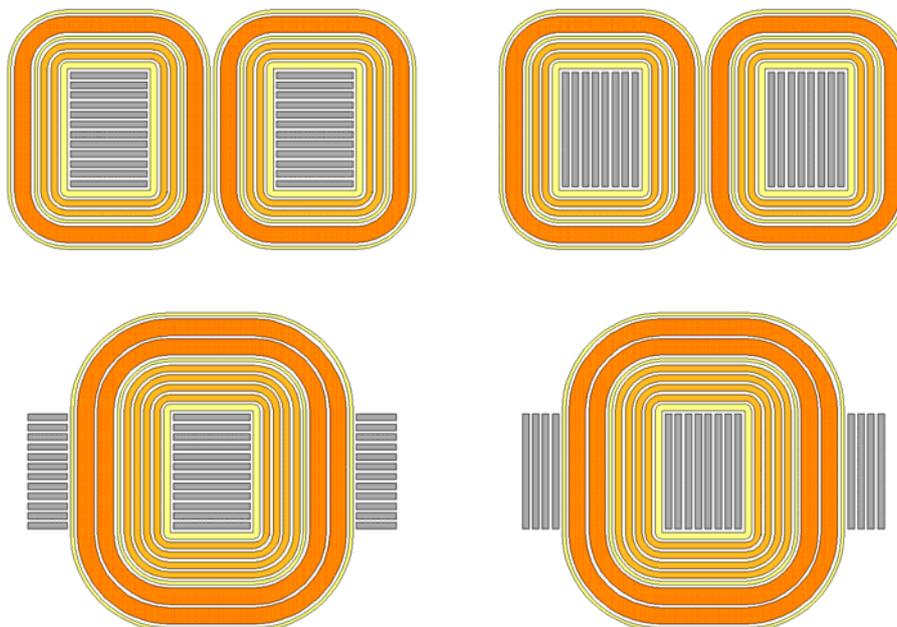


Figure 2.4 The cross-section of a transformer on the plane of electric current flow. The upper row indicates a core-type transformer and the lower row a shell-type of transformer. The transformers with planar laminations are shown in the left column and the wound-core transformers in the right column. The thinner cross-section of the winding in the core-type of transformers gives expectedly the advantage of the lower thermal stress.

## **2.2 Winding configuration**

The usual winding arrangement is that the low voltage coil has been wound closest to the core and the high-voltage coil has been wound on top of the low voltage coil, which is different that is shown in Figure 2.3 and Figure 2.4. A sandwiched configuration where the low voltage section is divided into two halves and the high voltage winding is placed between gives improved voltage regulation. Windings can be wound also in sections in side to side as a radial discs instead of axial layers. As a matter of fact the concentric coils reduce inefficient (leakage) area and thus improve the voltage regulation. One way or another there normally will be a considerable thickness of insulation between the primary and the secondary coil. The important in the winding configuration is the capability to dissipate heat and to bear electro-dynamical forces in the transient and short-circuit conditions, which tends to round out of the flat sides of the loop coil. In practice, most of the transformer coils are wound on a form, dipped in insulating varnish, backed in rigid mass, and then are assembled with core. The good thermal contact between the core and the winding is important in order to facilitate the heat dissipation.

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- Q.4 Why does the cooling problem become progressively more difficult as transformer size increases?
- Q.5 How should be the windings placed in order to take advantage of good backing and improved heat dissipation, lower leakage and better voltage regulation?
-

## 3 Design of a transformer

In order to design a transformer, or to examine in more detail how it departs from idealistic, it is necessary to understand how a transformer works, not just how to express its terminal relations in an approximate way. It is also important to know how the properties of the iron core affect the performance of the transformer. A real transformer becomes hot because of losses, and the output voltage may vary with load even when the primary voltage is held constant. It is important not simply to design a transformer that will do, but one that is economical, efficient and makes the best use of available materials.

Transformer *design consists* of five parts: physical understanding, mathematical modelling, synthesis, analysis and a cost functional [24]. Analysis generally means the action of taking something apart in order to study it. Synthesis is an opposite process which results in a new creation. Design as a process is originating and developing a plan for a new object. Design as a result is the final plan or proposal to produce the new object. Designing normally requires considering functional, and many other aspects of an object, which usually requires considerable research, thought, modelling, iterative adjustment, and re-design. Generally, functional refers to something able to fulfil its purpose or function. Design as a process can take many forms depending on the object being designed and the individual or individuals participating. Optimize the magnet device design. Optimized for a light-weight construction [11], cost, efficiency, reliability or some other requirements.

The *design process* can be detailed and broken down in a number of consequential steps or/and loops. By giving a shot overview, the size, shape and material of the core must be chosen, and the number of turns of the primary and secondary windings. The size of the wire and the insulation determines if the windings will fit in the space available. The windings must be arranged for minimum leakage flux. The insulation determines the permissible temperature rise, and means of cooling must balance the loss of energy under load. The number of primary turns is determined so that the magnetizing current is limited to an acceptable value, and depends on the length of the core. The area of the core is determined by the required volts per turn, now that the total number of turns is known. These are the things that determine the size and weight of a transformer. Design process is an iterative process that may include a number of sweeping statements or simplifying assumptions e.g. the flux density is uniform throughout the core.

The next lines give an overview of functionality parameters of a transformer, investigate the size of a transformer and describe the model of a transformer.

### 3.1 Functionality specification

Despite the many types of electronic transformers, their theory of operation does not differ. Electrical functions are usually similar but design characteristics can differ in certain ways. Some examples are; unipolar versus bipolar core utilization, saturating or not saturating, degree of energy storage, regulation, and transformer impedance.

The purpose of a transformer is a power conditioning where the voltage change-ratio and the galvanic separation between the source and the load are achieved. The transformer can stand alone for power conditioning or include more components in order to adapt the power source to the load. Apart from that the power conditioning circuit may include protection circuits or elements built into the transformer.

### Rated power

The apparent power of a transformer depends on the flow densities such as electric current, magnetic flux and heat loss power density [11]. Basically the size of transformer can be evaluated according the rated power, frequency and the dissipated losses.

### Rated primary and secondary voltage

The primary voltage and the power conditioning specify the number of turns in the windings.

### Regulation

Regulation includes both the input and the output voltage variation of the transformer. Line Regulation is the variation of main voltage, applied to the primary winding of the transformer. The changing of voltage on the primary winding results in a change of a voltage on secondary side with the same percent (which is by standard  $\pm 10\%$ ). Load Regulation is a change in output voltage with a change of load.

$$\delta U_2 = \frac{U_{20} - U_{2L}}{U_{2L}} \cdot 100\% \quad (3.1)$$

where the output voltage at no load is  $U_{20}$  and output voltage at full load is  $U_{2L}$ .

### Mounting and cooling conditions

The size of any electromagnetic or mechanic converter is basically limited by thermal conditions. An allowed dissipation power can be given or the cooling conditions can be specified. One of these conditions can specify the temperature in the transformer in respect to the generated heat losses. The hot spot temperature and the allowed temperature design is the actual limit.

### Insulation class

Insulation systems include all isolation materials used in manufacture of transformers such as bobbins, sheet film, film tape insulations, varnish covering of wires, varnish for impregnation etc. The temperature stability each component should not be lower than the temperature value for specific insulation classes. The other important function is the electrical insulation and galvanic separation.

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Q.6 Does output voltage  $U_2$  decreases proportionally with the increase of the load and why? How it depends on the size of a transformer?

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## 3.2 Geometric modelling

The proportions of the core-type and the shell-type transformers are investigated. The purposes of these investigations are

1. to determine theoretically the maximum flow cross-section areas for the electric and the magnetic circuit,
2. to express the size expression based to transparent power, volume and the electromagnetic flow densities,
3. to find out the proportions of the transformer when considering the heat losses and heat transfer i.e. the maximum temperature rise in a transformer.

## Geometry

The geometric parameterization of the shell-type and the core-type of transformer are visualized in Figure 3.1. As it is expressed the transformers should fit in to the same rectangular volume.

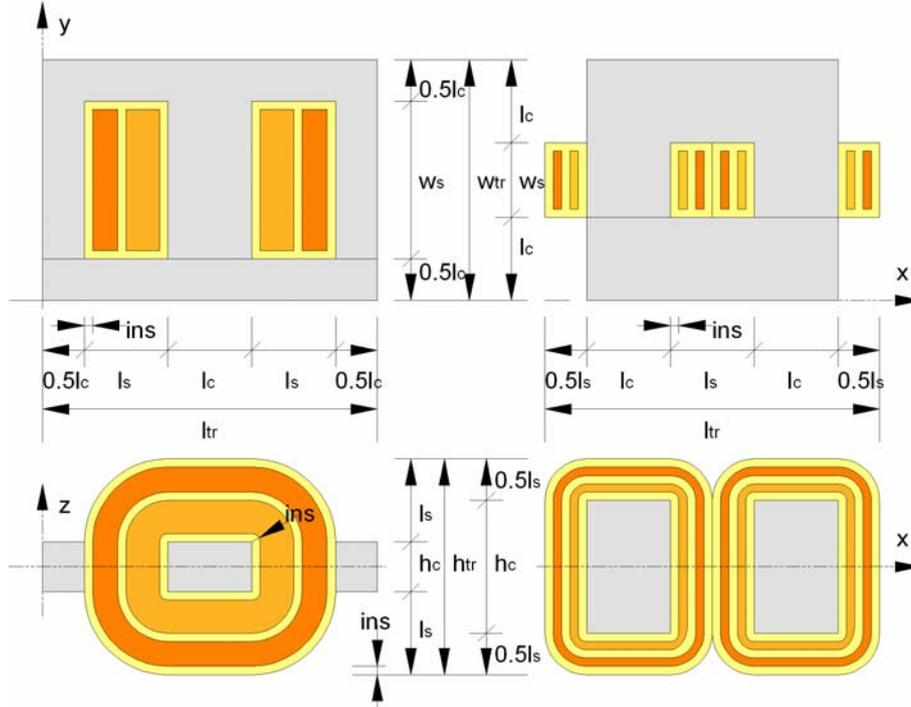


Figure 3.1 The geometry parameterization of the shell type (left column) and the core type (right column) of transformer. The upper row shows the magnetic flux flow plane ( $xy$ -plane) and the lower row shows the electric current flow plane ( $xz$ -plane).

The overall transformer length  $l_{tr}$ , width  $w_{tr}$  and height  $h_{tr}$  defines the bounding volume  $V_{tr}$  that is a volume of a rectangular parallelepiped box where the transformer fits in. The overall dimensions can be expressed through a single dimension and proportions to this chosen reference dimension such as relative width  $k_W$  and relative height  $k_H$  compared to the overall length  $l_{tr}$ .

$$V_{tr} = l_{tr} w_{tr} h_{tr} = l_{tr} \cdot k_W l_{tr} \cdot k_H l_{tr} = k_W k_H l_{tr}^3 \quad (3.2)$$

The length of the leg in the magnetic core is expressed by using the relative slot length  $k_s$  and the length needed to form a one electromagnetic pole. For the single-phase core and shell type of transformers the number of poles  $N_p=2$ .

$$l_c = \frac{l_{tr}}{N_p} (1 - k_s) \quad (3.3)$$

Similarly the length of the slot is formulated according to the length of a single electromagnetic pole and the relative slot length.

$$l_s = \frac{l_{tr}}{N_p} k_s \quad (3.4)$$

The width of the slot is given by the total width of the transformer  $w_{tr}$  minus the width of the magnetic core yokes. The width of the magnetic back-core equals to the length of the magnetic leg-core. Depending on the type of transformer core the width of the magnetic back core (yoke) is different and this is taken into consideration with the width factor of a transformer  $k_{TW}$ .

$$w_s = w_{tr} - k_{TW}l_c = k_W l_{tr} - k_{TW}l_c \quad (3.5)$$

The height of the magnetic core is given by the total height of the transformer  $h_{tr}$  minus the height of the ends of windings. The height of the ends of windings equals to the length of the slot.

Depending on the type of transformer core the height of the ends of windings is different and this is taken into consideration with the height factor of a transformer  $k_{TH}$ .

$$h_c = h_{tr} - k_{TH}l_s = k_H l_{tr} - k_{TH}l_s \quad (3.6)$$

The winding window or the slot is the available area inside the ferrous core where the winding wires into.

$$\begin{aligned} A_e &= k_i k_f l_s w_s = k_i k_f \frac{l_{tr}}{N_p} k_s \left( k_W l_{tr} - k_{TW} \frac{l_{tr}}{N_p} (1 - k_s) \right) = \\ &= k_i l_{tr}^2 \frac{k_f k_s}{N_p} \left( k_W - \frac{k_{TW} (1 - k_s)}{N_p} \right) \end{aligned} \quad (3.7)$$

The available cross-section area for the ferrous lamination stack is

$$\begin{aligned} A_m &= k_l l_c h_c = k_l \frac{l_{tr}}{N_p} (1 - k_s) \left( k_H l_{tr} - k_{TH} \frac{l_{tr}}{N_p} k_s \right) = \\ &= l_{tr}^2 \frac{k_l (1 - k_s)}{N_p} \left( k_H - \frac{k_{TH} k_s}{N_p} \right) \end{aligned} \quad (3.8)$$

The product of cross-section areas to the bounding volume of a transformer indicates the geometrical optimal relation between the electric and the magnetic core.

$$l_{tr} k_p = \frac{A_e A_m}{V_{tr}} = l_{tr} \frac{k_i k_f k_l (1 - k_s) k_s}{k_W k_H N_p^2} \left( k_W - \frac{k_{TW} (1 - k_s)}{N_p} \right) \left( k_H - \frac{k_{TH} k_s}{N_p} \right) \quad (3.9)$$

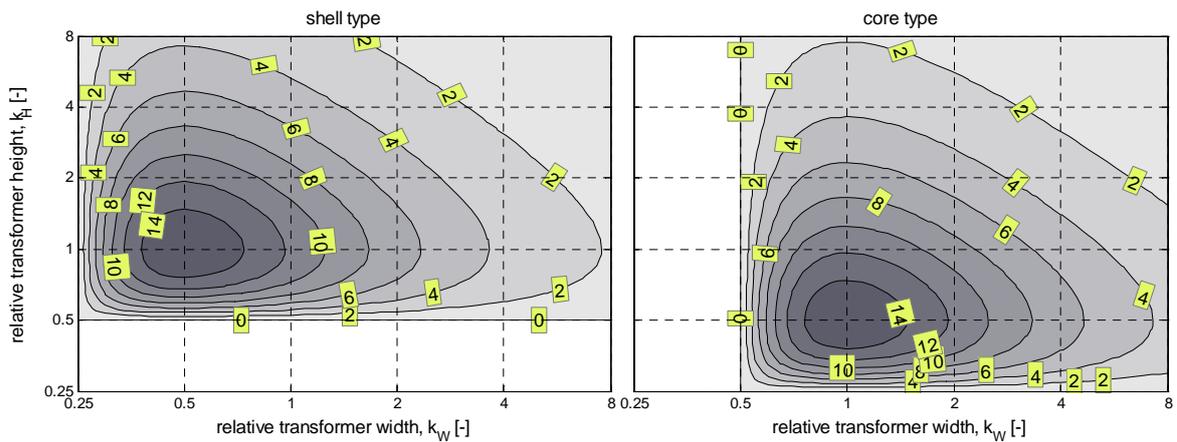


Figure 3.2 The geometrical optimum for a single-phase shell and core type of transformer by considering the constant envelope volume.

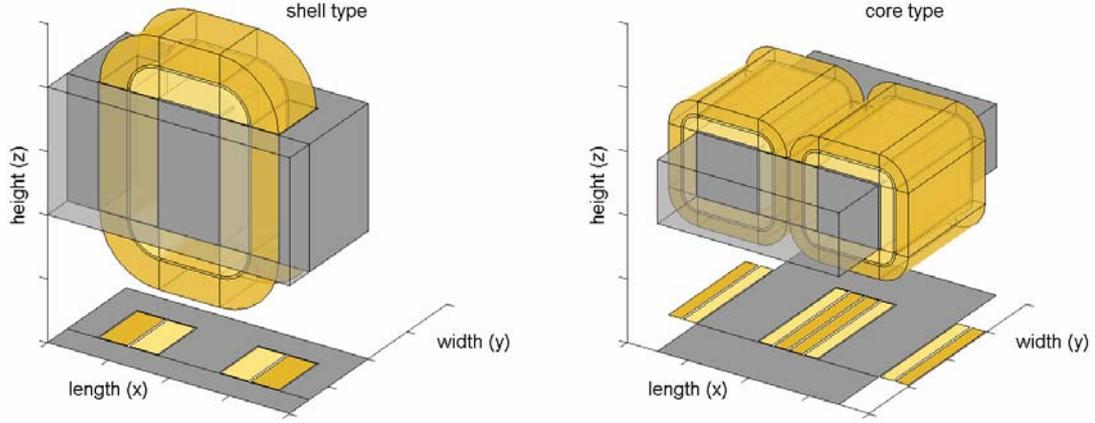


Figure 3.3 The geometry of a single-phase shell and core type of transformer at the point of the geometrical optimum.

Mean length of magnetic path for the shell-type and the core-type of transformer are

$$\begin{aligned}
 2\left(\frac{l_{tr}}{2} - 0.5l_c\right) + 2(w_{tr} - 0.5l_c) &= 2\left(\frac{l_{tr}}{2} - \frac{l_{tr}}{N_p}(1-k_s) + k_w l_{tr}\right) = \\
 &= 2l_{tr}\left(\frac{1}{2} + k_w - \frac{1-k_s}{N_p}\right)
 \end{aligned} \tag{3.10}$$

$$\begin{aligned}
 2(l_{tr} - l_s - l_c) + 2(w_{tr} - l_c) &= 2\left(l_{tr} - \frac{l_{tr}}{N_p}(1-k_s) - \frac{l_{tr}}{N_p}k_s + k_w l_{tr} - \frac{l_{tr}}{N_p}(1-k_s)\right) = \\
 &= 2l_{tr}\left(1 - \frac{2-k_s}{N_p} + k_w\right)
 \end{aligned} \tag{3.11}$$

Mean length of the winding for the shell-type and the core-type of transformer are

$$\begin{aligned}
 2(l_{tr} - l_s - l_c) + 2(h_{tr} - l_s) &= 2\left(l_{tr} - \frac{l_{tr}}{N_p}(1-k_s) - \frac{l_{tr}}{N_p}k_s + k_h l_{tr} - \frac{l_{tr}}{N_p}k_s\right) = \\
 &= 2l_{tr}\left(1 - \frac{1+k_s}{N_p} + k_h\right)
 \end{aligned} \tag{3.12}$$

$$\begin{aligned}
 2\left(\frac{l_{tr}}{2} - 0.5l_s\right) + 2(h_{tr} - 0.5l_s) &= 2\left(\frac{l_{tr}}{2} - \frac{l_{tr}}{N_p}k_s + k_h l_{tr}\right) = \\
 &= 2l_{tr}\left(\frac{1}{2} + k_h - \frac{k_s}{N_p}\right)
 \end{aligned} \tag{3.13}$$

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Q.7 What would be different if the geometric optimum would be derived for the three-phase transformer with different core types?

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### 3.3 Size equations

The size of transformer is related to the power of the electromagnetic device and the allowed flow densities such as current density, flux density and the loss density.

#### Ideal transformer

In an ideal transformer the magnetic coupling between the windings is perfect. There is the same core flux  $\phi(t)$  that links each turn of each winding. Apparent power of an ideal lossless transformer is expressed as

$$S = \frac{1}{2} U_m I_m \quad (3.14)$$

here is assumed sinusoidal variation of voltage and current and instead of rms values the peak values has been used instead. Considering the lossless electromagnetic circuit the voltage of the electric circuit can be directly linked to the induced back electromotive force (emf) and the magnetic flux in the magnetic circuit.

$$u(t) = U_m \cos(\omega t) = e(t) = \frac{d\psi(t)}{dt} = N \frac{d\phi(t)}{dt} \quad (3.15)$$

Ideally it is assumed that all the magnetic flux links with the winding and flux can be expressed directly from the voltage that is applied to the winding.

$$\phi(t) = \Phi_m \sin(\omega t) = \frac{U_m}{\omega N} \sin(\omega t) = B_m A_m \sin(\omega t) \quad (3.16)$$

The maximum value of magnetic flux  $\Phi_m$  is related to the cross-section area of the pure magnetic conductor  $A_m$  and the maximum flux density  $B_m$  that is defined by the material ability to conduct magnetic flux and the magnetic saturation  $B_{sat}$ . Similar to the magnetic circuit, the flow in the electric circuit is defined by current  $I_m$  (maximum value) in the single turn which is related to the total Ampere turns  $NI_m$  i.e. magnetomotive force (mmf), the cross-section area of the pure electric conductor  $A_e$  and the maximum current density  $J_m$  that is defined by device's ability to conduct heat flow and the thermal limit  $\mathcal{G}_{coil}$ .

$$i(t) = I_m \cos(\omega t + \varphi) = \frac{NI_m}{N} \cos(\omega t + \varphi) = \frac{J_m A_e}{N} \cos(\omega t + \varphi) \quad (3.17)$$

By substituting the maximum values of voltage and current in the equation of the apparent power the size of the transformer can be expressed through the nominal power and the magnetization frequency the allowed current and flux density.

$$S = \frac{1}{2} U_m I_m = \frac{1}{2} \omega B_m J_m A_e A_m \quad (3.18)$$

The specific power of transformer depends on the electric loading  $J_m$ , the magnetic loading  $B_m$  and the magnetization frequency  $\omega$  as well as the geometry relates dimension  $l_r$  and proportions  $k_p$ .

$$\frac{S}{V_{tr}} = \frac{1}{2} \omega B_m J_m l_r k_p \quad (3.19)$$

The size of the transformer can be smaller by increasing the loadings or magnetization frequency. These parameters are related to the power loss generation and therefore it is important to study the transformers ability to dissipate the heat.

## Considering losses and heat dissipation

The properties of media, where the flows take place, have not been considered so far. Apart from the electric current flow and the magnetic flux flow there is also a heat flow that is caused by power loss sources. The power loss sources are related to the electrical and magnetic loadings and the material properties. The conductor loss for the direct current  $P_{cu,dc}$  is expressed through the power loss density, which depends on resistivity  $\rho$  and the current density square  $J^2$ , and the volume of the conductor  $V_e$ .

$$P_{cu,dc} = I^2 R = \left( \frac{JA_e}{N} \right)^2 \rho \frac{l_e N}{A_e} = \rho J^2 A_e l_e = \frac{1}{2} \rho J_m^2 V_e \quad (3.20)$$

The remagnetization loss in the magnetic conductor for the symmetric sinusoidal excitation can be found from the specific loss data  $k_{fe}$  and core volume  $V_m$

$$P_{fe} = k_{fe} (B_m, \omega) V_m \quad (3.21)$$

Alternatively, the total core loss can be found from the loss separation formula, where the hysteresis, eddy current and anomalous loss coefficients  $k_{hy}$ ,  $b$ ,  $k_{ec}$ ,  $k_{an}$  complete the expression.

$$P_{fe} = (k_{hy} B_m^h \omega + k_{ec} B_m^2 \omega^2 + k_{an} B_m^{1.5} \omega^{1.5}) V_m \quad (3.22)$$

The relation between the flow density and the loss density in an electric conductor can be expressed as

$$q_e = \frac{1}{2} J_m^2 \rho \quad (3.23)$$

and the approximate relation in the magnetic conductor, where  $k_m$  indicates the loss coefficient.

$$q_m = \frac{1}{2} B_m^2 \omega^2 k_m \quad (3.24)$$

According to the heat transfer there is a balance between the heat sources and the heat dissipation. The steady state heat distribution is defined by the Poisson partial differential equation,

$$\lambda_x \frac{d^2}{dx^2} \mathcal{G} + \lambda_y \frac{d^2}{dy^2} \mathcal{G} + \lambda_z \frac{d^2}{dz^2} \mathcal{G} + q = 0 \quad (3.25)$$

where thermal conductivities  $\lambda_x$ ,  $\lambda_y$  and  $\lambda_z$  define the heat flow from a region with higher temperature  $\mathcal{G}$  to a region with lower temperature, respectively, in  $x$ ,  $y$  and  $z$  direction according to the spatial heat source distribution  $q$  [ $W/m^3$ ] and predefined dissipation. In case of one-dimensional (1D) heat distribution the heat is generated within the material itself is

$$\lambda_x \frac{d^2}{dx^2} \mathcal{G} + q = 0 \quad (3.26)$$

By applying the boundary conditions that there are no heat flow  $Q$  [ $W/m^2$ ] through the symmetric surface(s) and the predefined temperature of the bounding surfaces is  $\mathcal{G}_s$ , then the temperature distribution in a plate with a thickness of  $d=2x_s$ , and the reference temperature  $\mathcal{G}(x_s)=\mathcal{G}_s$

$$\mathcal{G}(x) = \mathcal{G}(x_s) + \frac{q}{2\lambda_x} \cdot (x_s^2 - x^2) \quad (3.27)$$

And in a cylinder with a diameter of  $d=2r_s$ , and the reference temperature  $\mathcal{G}(r_s)=\mathcal{G}_s$

$$\vartheta(r) = \vartheta(r_s) + \frac{q}{4\lambda_r} \cdot (r_s^2 - r^2) \quad (3.28)$$

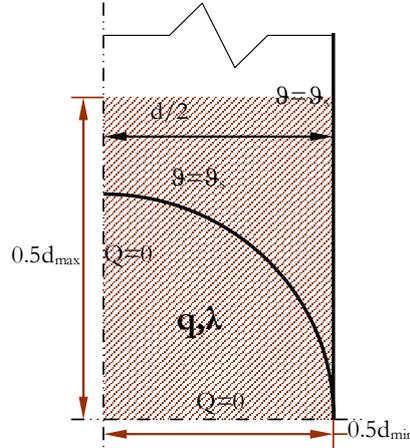


Figure 3.4 The symmetric part of a conductor cross-section with an internal heat generation  $q$  and an isotropic thermal conductivity  $\lambda$ . The symmetry planes with predefined heat flux  $Q$  are the same for a cylinder and a plate.

The highest temperature rise  $\Delta\vartheta$  in the centre of a plate or a cylinder, which thickness or diameter, respectively is denoted with  $d$  and the geometry related coefficient  $k_g$  is 1 for a plate and 2 for a cylinder.

$$\Delta\vartheta = \frac{q}{2k_g \lambda} \frac{d^2}{4} \quad (3.29)$$

The realistic cross-section area of an electric conductor or a magnetic conductor is between the rectangle and the circle, which corresponds to the striped area in Figure 3.4. The geometric shape and proportions can be taken account with the geometry related coefficient that is found empirically from the ratio of the shortest dimension to the longest dimension of the cross-section area.

$$k_g = 1 + \left( \frac{d_{\min}}{d_{\max}} \right)^2 \quad (3.30)$$

By choosing that the electric circuit produces the same amount of losses than the magnetic circuit, and the total temperature rise in the electromagnetic device is the sum of the temperature rise in the electric circuit and in the magnetic circuit.

$$\Delta\vartheta = \frac{q}{8} \left[ \frac{\min(l_s, w_s)^2}{\lambda_e \left( 1 + \left( \frac{\min(l_s, w_s)}{\max(l_s, w_s)} \right)^2 \right)} + \frac{\min(l_c, h_c)^2}{\lambda_m \left( 1 + \left( \frac{\min(l_c, h_c)}{\max(l_c, h_c)} \right)^2 \right)} \right] = qk_g \quad (3.31)$$

Even though the thermal coupling between the electric and the magnetic circuits is complex, this simple approach helps to choose the geometrical proportions of a transformer in order to reduce the maximum temperature rise. A narrow and high cross-section of a circuit has a smaller temperature rise than a square shaped cross-section with the same area. Therefore the results

(Figure 3.5) indicate the sum of the thermal loadings in the electric circuit and in the magnetic circuit that have their maximum thermal resistance at the different geometrical proportions of a transformer. In the sake of comparability the thermal conductivity is taken the same  $\lambda_c = \lambda_m = 1 \text{ W/mK}$ . The maximum thermal resistance for the electric circuit is in region  $k_H = 0.5$   $k_W = 0.7$  – shell and  $k_H = 0.25$   $k_W = 1.1$  – core, and for the magnetic circuit is in region  $k_H = 1.1$   $k_W = 0.25$  – shell and  $k_H = 0.7$   $k_W = 0.5$  – core. The thermal resistance is calculated in respect with the transformer envelope volume that is  $V_{tr} = 1 \text{ m}^3$ .

$$R_g = \frac{k_g}{V_{tr}} \quad (3.32)$$

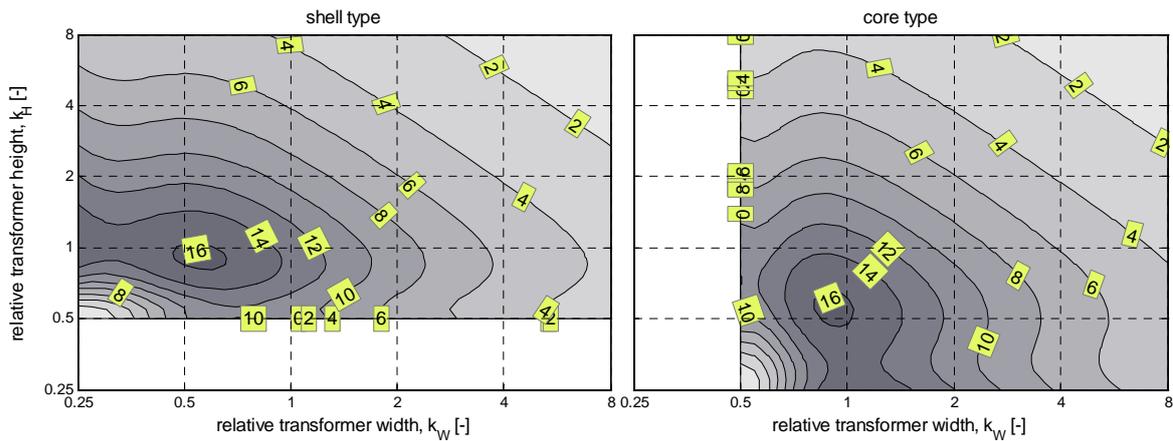


Figure 3.5 Thermal resistance  $R_g \cdot 1000$  in respect with type of transformer and proportions of transformer geometry.

---

Q.8 How could be understood Faraday's law, Lenz law and the definition and sign of the inductance element in circuit theory according to Kirchhoff's law?

Q.9 How could be understood the fundamental transformer equation

$$U_{rms} = 4.44 N f \phi_{max}$$

and the equation derived above?

---

### 3.4 Equivalent circuit of a transformer

Equivalent circuit facilitates to analyze the electromagnetic behaviour of the transformer.

#### Ideal transformer

Ideal describes the ideal electromagnetic coupling with no losses or voltage/mmF drop in the electric/magnetic circuits. The electrical equations in the ideal transformer are

$$\begin{cases} u_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt} \\ u_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\phi}{dt} \end{cases} \quad (3.33)$$

The magnetic equations

$$\begin{cases} \phi = PN_1i_1 \\ \phi = PN_2i_2 \end{cases} \quad (3.34)$$

where for the ideally coupled windings the primary mmf balances the secondary mmf. The mutual flux  $\phi$  is the means of transfer of energy from primary to secondary, and links both windings. In an ideal transformer, this flux requires negligibly small ampere-turns to produce it, so the net ampere-turns, primary plus secondary, is about zero. When a current is drawn from the secondary in the positive direction, ampere-turns decrease substantially. This must be matched by an equal increase in primary ampere-turns, which is caused by an increase in the current entering the primary in the positive direction. In this way, the back-emf of the primary (the voltage induced in it by the flux  $\phi$ ) equals the voltage applied to the primary, as it must. This fundamental explanation of the operation of a transformer must be clearly understood before proceeding further. The transformation coefficient  $n$  is in accordance with instantaneous power balance

$$n = \frac{u_2}{u_1} = \frac{N_2}{N_1} = \frac{i_1}{i_2} \quad (3.35)$$

If an impedance  $Z_2$  is connected to the secondary then  $U_2/I_2=Z_2$  and considering the relations between the primary and the secondary voltages and currents then the impedance  $Z_1$  by the primary terminals is

$$Z_1 = \frac{U_1}{I_1} = \frac{1}{n^2} \frac{U_2}{I_2} = \frac{1}{n^2} Z_2 \quad (3.36)$$

that is the transformer property of changing impedances.

## Real transformer

In the real transformer, the winding resistances are not zero, the magnetic coupling between the coils is not perfect and the reluctance of the magnetic core is not zero. Including the imperfections to the formulation of the ideal transformer the electrical equation becomes

$$\begin{cases} u_1 = R_1i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ u_2 = -R_2i_2 - L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{cases} \quad (3.37)$$

magnetic equations

$$\begin{cases} \phi_1 = \phi_{\sigma 1} + \phi_{21} = \frac{L_{\sigma 1}i_1}{N_1} + \frac{Mi_2}{N_2} = \frac{L_1i_1}{N_1} \\ \phi_2 = \phi_{\sigma 2} + \phi_{12} = \frac{L_{\sigma 2}i_2}{N_2} + \frac{Mi_1}{N_1} = \frac{L_2i_2}{N_2} \end{cases} \quad (3.38)$$

This treats the windings as a pair of mutually coupled coils with both primary and secondary windings passing currents. The negative sign in these equations arise from the reversed direction of the secondary current  $i_2$ . From the transformer equation, the primary MMF must equal the secondary MMF, and since these are in opposite directions, in an ideal transformer they cancel so that there is no overall resultant flux in the core. That this is so can be seen by realising that any unopposed primary emf would create a large primary current and therefore a large flux in the core due to the primary winding. However, this large flux would necessarily cause a large current to flow

in the secondary circuit and this current must create an opposing flux that effectively cancels the initiating primary flux. In a non-ideal transformer, the resultant flux in the core is that needed to magnetise the core. This is called the magnetising flux. Apart from the coupling flux the total flux linkage of the coil includes also a leakage flux that does not couple with the secondary coil. By describing the flux current elements as inductances the imperfection of the real magnetic coupling can be taken into the consideration.

$$\begin{cases} L_1 = L_{\sigma 1} + \frac{N_1}{N_2} M \\ L_2 = L_{\sigma 2} + \frac{N_2}{N_1} M \end{cases} \quad (3.39)$$

after the substitutions of the ‘description’ of magnetic circuit into the electrical equations the system equation becomes

$$\begin{cases} u_1 = R_1 i_1 + L_{\sigma 1} \frac{di_1}{dt} - \frac{1}{n} M \frac{d}{dt} (i_1 - ni_2) \\ u_2 = -R_2 i_2 - L_{\sigma 2} \frac{di_2}{dt} + M \frac{d}{dt} (i_1 - ni_2) \end{cases} \quad (3.40)$$

and the resulting equivalent circuit is shown in Figure 3.6.

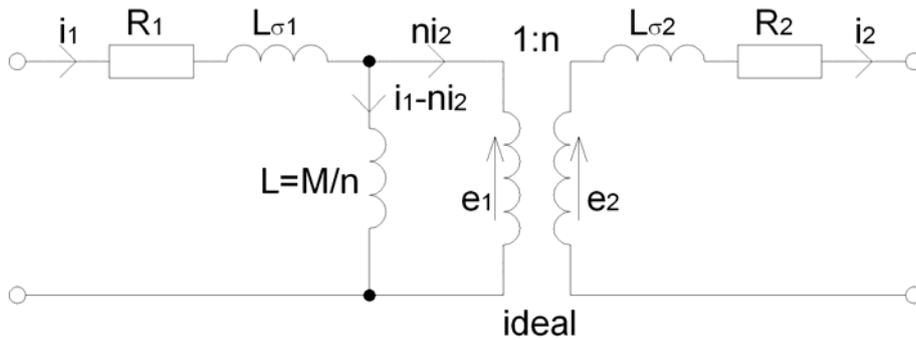


Figure 3.6 Circuit representation for the equation system of a transformer

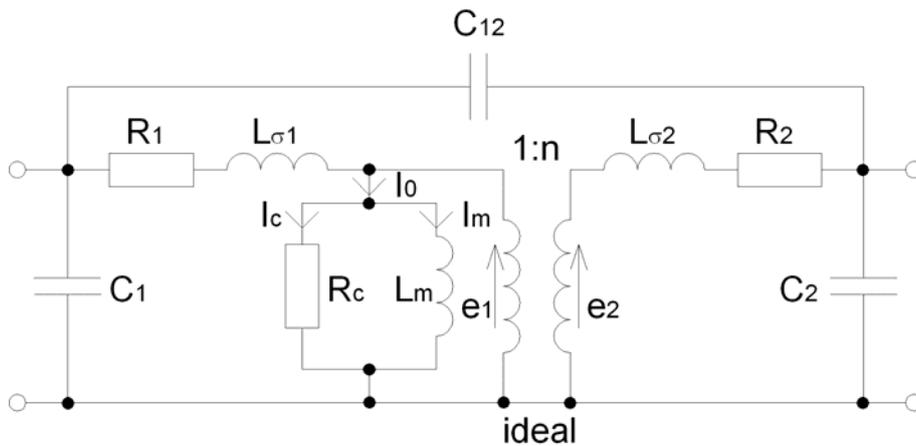


Figure 3.7 Complete equivalent circuit for a transformer includes the effects of shunt and inter-winding capacitance, stray inductance, magnetic loss, and winding resistance. Under low frequency or large-signal conditions, the shunt primary inductance can become nonlinear if the transformer is driven into saturation.

Figure 3.7 indicates the complete equivalent circuit that is equally applicable to pulse and wideband transformers [2][10]. This results that the circuit can be used in the analysis that focuses rather on time domain (pulse transformers) rather than frequency domain (band transformers). The particular difference between Figure 3.6 and Figure 3.7 is that the first circuit neglects the capacitance effects between the turns and between the windings, magnetic non-linearity is ignored and iron losses are ignored. For a given transformer, the values of many equivalent circuit elements are same for pulse and wideband application. The circuit elements determine the electromagnetic behavior of the transformers. Description of the circuit parameters:

- Nonzero resistance of the windings (R1 and R2) power dissipation through heating, and impedance transformation,
- Frequency dependence of the material permeability  $\mu_r=f(\text{freq})$ ,
- Magnetic losses (Rc), due to material opposition (friction loss) to orient domain walls according to the external magnetizing field or due to material opposition (conductive loss) to conduct current, which is caused by the induced voltage in the alternating magnetic field.
- Intra-winding capacitance (turn to turn within a winding—C1 C2), Inter-winding capacitance (primary to secondary—C12) Parasitic capacitances limit the upper bandwidth of operation and also reduce the isolation the transformer can provide
- Finite primary winding inductance (Lm), For large signal operation, the core will saturate, and the inductance will change during the course of a voltage cycle. This causes non-linear behavior, and can lead to catastrophic transformer failure. It is important to realize that the saturation problem is a function of the applied voltage and frequency only.
- Finite flux capability of the core material, leading to saturation (non-linear behavior of Lm)
- Leakage inductance of the windings (L1 and L2)
- The equivalent circuit consists of an ideal transformer of ration 1:n that ideally establish the mutual flux,

In the ideal transformer the magnetization is lossless

$$E_1 = \omega N_1 \Phi \quad (3.41)$$

in the real transformer the magnetic core has a mmf-drop that drags a magnetizing current

$$I_{0m} = \frac{\omega N_1}{X_m} \Phi \quad (3.42)$$

In addition the magnetization process causes losses due to magnetic hysteresis in the materials and induced electromotive force that is reason for the eddy currents in a conductive material.

$$I_{0c} = \frac{\omega N_1}{R_c} \Phi = \frac{P_{fe}}{\omega N_1 \Phi} \quad (3.43)$$

The equivalent circuit can be further simplified by allowing small calculation inaccuracies without significantly increasing the computation error.

- The small voltage drop in R1 and X1 compared to U1, and Small magnetizing current I0 in comparison with load current I1 allows the shunt terminals transfer to primary terminals,
- The secondary quantities R2 and X2 may be replaced on the primary side by using ideal transformer property,

- The new equivalent circuit can be used for small power transformer that simplifies the analysis and parameter identification from the experiments.

### Phasor diagram

With steady state a.c. conditions the instantaneous values can be represent with phasors in phasor-diagrams (frequency domain). The best way to understand things is to draw a phasor diagram of the currents, voltages and fluxes. In doing this, we draw the diagram in stages, showing how to proceed step by step so that it would be possible to make accurate numerical estimates of each quantity. The primary current is given by magnetizing current and the referred secondary current for a loaded transformer:

$$I_1 = I_0 + nI_2 \quad (3.44)$$

The secondary voltage can be expressed from the equivalent circuit

$$U_2 = E_2 - I_2(R_2 + jX_2) = nU_1 - nI_1(R_1 + jX_1) - I_2(R_2 + jX_2) \quad (3.45)$$

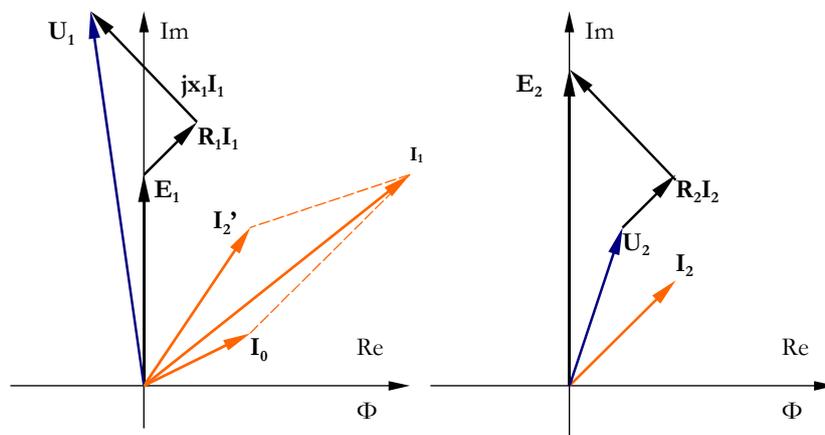


Figure 3.8 Phasor diagrams of the operating frequency domain for the primary side (on the left) and for the secondary side (on the right).

The design goal is to design a transformer as close to an ideal transformer as possible. Therefore some of the transformers such as measuring instrument transformers need to be carefully designed.



## 4 Numerical field modelling of a transformer

Most of the Finite Element computational software for electromagnetism considers *current-driven* magnetic problems. However, the resistance and the linking flux of the current carrying coil can be evaluated, which in turn is the basis to estimate the voltage across the winding. In the case of determining of an operating point of a transformer a voltage across the primary and the secondary load impedance of the transformer is the input to iteratively determine the currents required to produce specified terminal voltages with arbitrarily specified external loads. Apart from the magnetic problem the thermal problem is present. According to the cooling conditions and the hot-spot temperature of the maximum allowed currents can be evaluated. When using FEMM for 2D FE solver to compute 3D problem in electromagnetism the following effects can be considered:

- The effect of the small air gaps at lamination joints,
- Rolling direction by introducing anisotropic material,
- End turn impedance,
- Hysteresis and eddy current loss in the laminations,
- Proximity and skin effects in the transformer windings

A simple transformer model has been used to visualize the example of determining the operation point of transformer at an open-circuit, a nominal loading and also a short-circuit condition. The static model in electromagnetism is established i.e. all the reference and calculated values can be seen as the maximum values. This mistake is allowed in order to facilitate the understanding of FE modelling.

### 4.1 Model parameterization

The example transformer consists of a primary winding (orange rectangles in the left) and a secondary winding (yellow rectangles) that occupy the slot of the UI or LL transformer core (a grey square with an opening). The height of the square shaped iron core is 40 mm and this dimension is kept constant while the width of the core sides is to be changed in order to find the optimal proportions between the iron core and the windings (Figure 4.1).

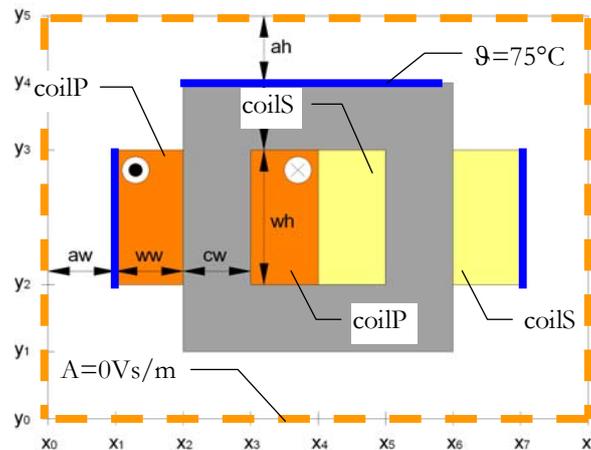


Figure 4.1 FE model setup for solving the problem of heat transfer (with Mirage) and electromagnetism (with FEMM)

The initial dimensions are the width of the winding  $w_w=10\text{ mm}$  and the height of the winding  $w_h=20\text{ mm}$ , which define the cross-section area of the primary and the secondary windings. The width of the iron core is  $c_w=10\text{ mm}$  and the region around the transformer is defined by two dimensions  $a_w=20\text{ mm}$  and  $a_b=20\text{ mm}$ . These last dimensions are necessary for Finite Element Modelling of the electromagnetic problem. The length of the transformer along the z-axis is  $c_l=40\text{ mm}$ .

Apart from the geometry the necessary information is the copper fill-factor of the winding  $K_f=0.6$ , that specifies the equivalent thermal conductivity  $\lambda_{eqv}=0.2\text{ W/Km}$  and the iron lamination that has iron fill-factor of  $\lambda_{mf}=0.95$  and the thickness of the lamination  $\lambda_{md}=0.5\text{ mm}$  that has the thermal conductivity of  $\lambda_{fe}=24\text{ W/Km}$ . The maximum allowed temperature in the winding is  $\vartheta_{coil}=135^\circ\text{C}$  and the temperature rise in the device is  $\Delta\vartheta=60^\circ\text{C}$ . The number of turns in the primary winding is  $N_1=1\text{ turn}$  and the turn ratio is  $N_1/N_2=1$ . Therefore total ampere turns and voltage per turn are considered. The initial current density is  $2\text{ A/mm}^2$  and this does not correspond always to the thermal condition therefore the current density has to be found iteratively from the thermal model that consider the geometrical change and the copper and the core losses in the loaded transformer. The specific copper losses are found in respect with current density square, temperature dependent resistivity and the copper fill factor. The iron losses that can be taken into consideration with the lag angle between the magnetic field intensity  $H$  and the flux density  $B$  and core conductivity are introduced. In this example the frequency is set  $freq=0\text{ Hz}$  and the core losses are estimated from the polynomial coefficient that fit the material characteristic (Figure 4.2)

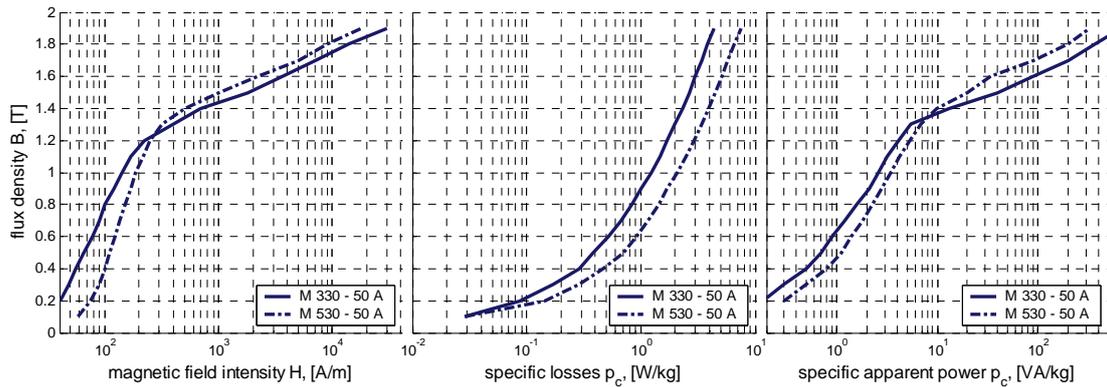


Figure 4.2 Characteristics of the electromagnetic steel type, which are used in the simulation and prototyping.

In the example model the chosen type of electromagnetic steel is M 530 – 50 A.

In the next lines the example of the main model parameterization is given. The text can be copied into lua file, and can be executed together with additional sub-files in order to solve a problem on heat transfer or electromagnetism. The variable that makes the choice between heat transfer problem setup (that is solved in Mirage) and the task on electromagnetism, which is solved in FEMM is `ask_fem`. Depending on the switch `0` or `1` the main lua script suppose to be executed in FEMM or Mirage, respectively.

A main command routine can be made that makes selection between the software and the problem to be solved.

```

-- magnetic and thermal design of a transformer

-- selection
ask_fem=1; -- 1-heat transfer, 0-electromagnetism
-- dofile("tmp_select_1.lua"); -- read in from a file that is created before

-- device parameterization
Ulp=230*sqrt(2); -- primary voltage max value
Bcm=1.5; -- initial flux density max value
Jcm=2; -- initial current density max value
Tcoil=135; -- max coil temperature
DeltaT=60; -- temperature rise in the device
Kf=0.6; -- copper fill factor in the winding
rcu0=2.4e-8; -- copper resistivity at 20C
rcu=rcu0*(1+0.004*(Tcoil-20)); -- copper resistivity at Tcoil
N1=1; -- number of turns in the primary side
Nratio=1; -- turn ratio in the transformer
lam_d=0.5; -- thickness of lamination
lam_f=0.95; -- lamination fill factor
lam_loss=15; -- loss angle
labmda_eqv=0.2; -- equivalent thermal conductivity of a coil
labmda_fe=24; -- thermal conductivity of a iron core
labmda_air=0.03; -- thermal conductivity of air
freq=0; -- frequency
pfe=(2.5517*Bcm^2-1.2936*Bcm+0.8143)*7700 -- core loss density W/m3 at 50Hz

-- geometry parameterization
cl=40e-3; -- length of core in z-axis
ww=10e-3; -- width of the winding
wh=20e-3; -- height of the winding
cw=10e-3; -- width of the iron core
aw=20e-3; -- width of air region from the winding
ah=20e-3; -- height of air region from the core

if ask_fem==0 then
    h=openfile("tmp_magn_res.txt","w") -- clear content of the output file
    closefile(h);
end
if ask_fem==1 then
    h=openfile("tmp_heat_res.txt","w") -- clear content of the output file
    closefile(h);
end

-- for kh = 1,9,1 do -- multiple step
for kh = 5,5,1 do -- single step

    -- geometric change
    ww=kh*2e-3; -- width of the winding
    wh=kh*4e-3; -- height of the winding
    cw=20e-3-ww; -- width of the iron core

    -- magnetization (idling transformer)
    if ask_fem==0 then
        filename=format('%s%g%s','tmp_trafo_B_',kh,'.bmp') -- format filename
        dofile("PTC_magn_1.lua"); -- execute a lua script
        mo_close(); mi_close(); -- close pre and post-processor
    end

    -- temperature limit of the loaded transformer
    if ask_fem==1 then
        filename=format('%s%g%s','tmp_trafo_T_',kh,'.bmp') -- format filename
        dofile("PTC_heat_1.lua"); -- execute a lua script
        ho_close(); hi_close(); -- close pre and post-processor
    end
end
end

```

- Q.10 The core loss model in the example bases on polynomial fit of the loss characteristic at a single frequency, how do consider harmonic losses? Are the core losses over or underestimated if they are calculated according to superposition of flux density frequency spectrum for the saturated core?
- Q.11 What is the purpose of specific apparent power characteristics and how are these connected to magnetization characteristics?

## 4.2 Magnetization

This sub-section describes an iterative model to estimate magnetization current of a transformer/an inductor when the target flux density is given. First a theoretical introduction will be given, that is later followed with an undemanding example, which is the sub-file *PTC\_magn\_1.lua* that is called from the model parameterization file.

Transformer can be seen as an electromagnetic inductor that is loaded with a secondary winding. Therefore, when the secondary loading is absent the magnetization process of the transformer can be studied. When considering an ideal electromagnetic inductor then the losses in the electric circuit and in the magnetic circuit can be neglected. According to Ampere's and Faraday's law the current carrying conductor causes magnetic field and alternating magnetic field induces an opposing electromagnetic force, respectively. For pure inductance the applied voltage equals to the transformed induced electromotive force and the latter is  $\pi/2$  ahead of the magnetic flux, the flux linkage and the current. The magnetic flux depends on magnetic permeance of an average flux path and the magnetomotive force.

$$Hl = \frac{B}{\mu}l = \phi \frac{l}{\mu A} = \phi R = \phi \frac{1}{P} = NI \quad (4.1)$$

The corresponding magnetic equivalent circuit including core, leakage and gap permeances is expressed in Figure 4.3. The sum of the magnetizing flux  $\phi_c$  in the core and the leakage flux  $\phi_\sigma$  define the total flux  $\phi$ .

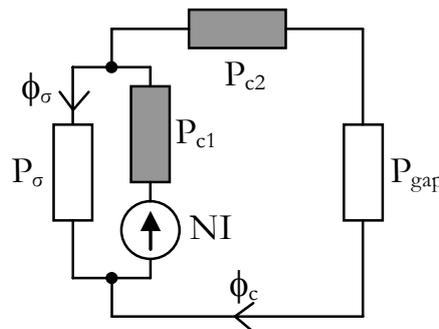


Figure 4.3 The magnetic equivalent circuit of an electromagnetic inductor. The magnetic core is divided into two parts: one which is enclosed with winding and the rest. The gap permeance indicates the total distance between the laminates on the same layer.

The nonlinear permeance,  $P_{c1}$ , indicates the electromagnetic core that is enclosed with the winding. The rest of the magnetic circuit is noted with  $P_{c2}$ . The magnetic/mechanical contact between the laminations at the same plane may include a small air-gap that can be taken into consideration with  $P_{gap}$ . The linear leakage,  $P_\sigma$ , can be geometrically derived as it is shown for the symmetric part of the winding/core cross-section in Figure 4.4.

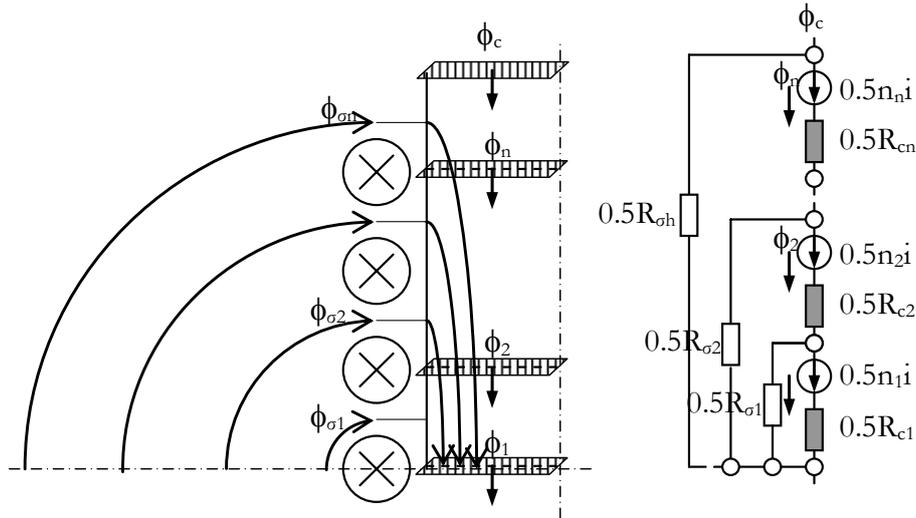


Figure 4.4 A symmetric part of a coil on an iron core and the corresponding equivalent circuit.

When the magnetic material BH curve has hysteresis, the fundamental components of vectors H and B are out of phase. In any case the induced voltage is  $\pi/2$  ahead of the flux, but due to losses a current component appears that is in phase with induced voltage and the corresponding active power covers the hysteresis loss in the magnetic material. Therefore hysteresis loss can be introduced by defining an angle between the H and B vectors, which is the case in FEMM AC problem of electromagnetism. In case of alternating magnetic field and the conductive magnetic core the induced electromagnetic force causes currents. These eddy currents cause opposing field and losses. The following assumption can be made in order to simplify the further calculations:

- Full penetration of magnetic flux,
- The magnitude of magnetic flux density is independent of the core loss mechanisms and depends only on magnetizing current,
- Uniform distribution of magnetic flux in order to facilitate core loss estimation.

As the geometry of the transformer is symmetrical, all the parameters, which are introduced for the electrical equivalent circuit (Figure 3.6), can be obtained. Instead of voltages, currents, inductances, etc the voltage per turn, the total current, the magnetic permeances ( $L/N^2$ ) can be introduced instead. This facilitates to keep thinking rather in cross-section areas than a number of turns. The corresponding electrical equivalent circuit is shown in Figure 4.5.

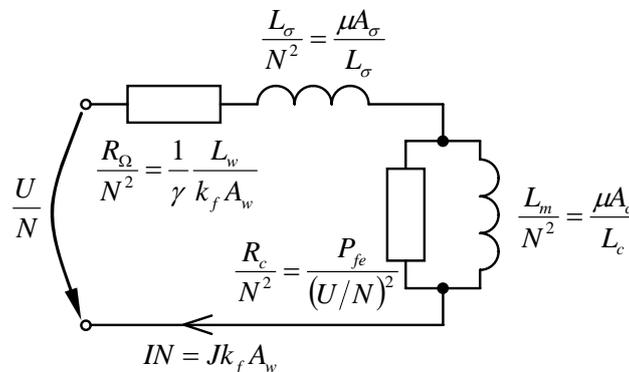


Figure 4.5 The electrical equivalent circuit of an inductor where the voltage and currents are represented in voltage per turns and ampere turns.

The electrical equation of the circuit (Figure 4.5) is

$$\frac{U}{N} = JA_w k_f \left( \frac{L_w}{\gamma k_f A_w} + j\omega \frac{\mu A_\sigma}{L_\sigma} + \frac{j\omega \frac{\mu A_\sigma}{L_\sigma} \frac{P_{fe}}{(U/N)^2}}{\frac{P_{fe}}{(U/N)^2} + j\omega \frac{\mu A_c}{L_c}} \right) \quad (4.2)$$

Instead of constant voltage the magnetizing current density is found iteratively with respect to constant flux density. The lua script *PTC\_magn\_1.lua* consists of a transformer geometry setup, a material characterization, a FE model setup, which includes boundary conditions and source definitions, solver and result analysis. The post-processing routine calculates iteratively the magnetization current density that grounds the desired flux density in the core. The stability and converging speed is determined by *weight* variable.

```
-- solving a electromagnetism problem with FEMM 4.0

x0=0; y0=0; -- origin and the rest of the coordinates
x1=x0+aw; x2=x1+ww; x3=x2+cw; x4=x3+ww;
x5=x4+ww; x6=x5+cw; x7=x6+ww; x8=x7+aw;
y1=y0+ah; y2=y1+cw; y3=y2+wh; y4=y3+cw; y5=y4+ah;
xi=1e-4; yi=1e-4; -- displacement increment

-- additional parameters
ElemSize=0.001; -- max finite element size

-- parameter calculation
Aw = ww*wh; -- winding cross section
N2 = ceil(N1*Nratio); -- rounds the value to the nearest integer
towards infinity
I1m = (Jcm*1e+6)*Aw*Kf; -- max value of the primary current
I2m = 0; -- max value of the secondary current
d1 = sqrt(4/pi*Aw*Kf/N1); -- diameter of primary conductor
d2 = sqrt(4/pi*Aw*Kf/N2); -- diameter of secondary conductor

-- create a new preprocessor for a magnetics problem
create(0)

-- problem definition
mi_probdef(freq,"meters","planar",1e-8,c1)

-- boundary definitions
mi_addboundprop("tang",0,0,0,0,0,0,0,0,0) -- 'zero' magnetic potential

-- material definitions
mi_addmaterial("core",500,500,0,0,0,2,lam_d,lam_loss,lam_f,0)
mi_addmaterial("ins",1,1,0,0,0,0.002,0,0,1,0)
mi_addmaterial("air",1,1,0,0,0,0,0,0,1,0)
--mi_addmaterial("coilP",1,1,0,0,0,1/rcu,0,0,1,4,0,0,1,d1*1000)
--mi_addmaterial("coils",1,1,0,0,0,1/rcu,0,0,1,4,0,0,1,d2*1000)
mi_addmaterial("coilP",1,1,0,0,0,1/rcu,0,0,1,0)
mi_addmaterial("coils",1,1,0,0,0,1/rcu,0,0,1,0)

-- circuit definition
mi_addcircprop("coilP",I1m,0,1)
mi_addcircprop("coils",I2m,0,1)

-- definition of non-linear magnetic characteristics
mi_addbhpoint("core",0,0)
mi_addbhpoint("core",0.03,20.0)
mi_addbhpoint("core",0.09,40.0)
mi_addbhpoint("core",0.21,60.0)
mi_addbhpoint("core",0.42,80.0)
```

```

mi_addbhpnt("core",0.69,100.0)
mi_addbhpnt("core",0.92,120.0)
mi_addbhpnt("core",1.06,140.0)
mi_addbhpnt("core",1.18,160.0)
mi_addbhpnt("core",1.26,180.0)
mi_addbhpnt("core",1.32,200.0)
mi_addbhpnt("core",1.35,220.0)
mi_addbhpnt("core",1.37,240.0)
mi_addbhpnt("core",1.39,260.0)
mi_addbhpnt("core",1.41,280.0)
mi_addbhpnt("core",1.43,300.0)
mi_addbhpnt("core",1.49,400.0)
mi_addbhpnt("core",1.55,600.0)
mi_addbhpnt("core",1.58,800.0)
mi_addbhpnt("core",1.60,1000.0)
mi_addbhpnt("core",1.62,1200.0)
mi_addbhpnt("core",1.64,1400.0)
mi_addbhpnt("core",1.66,1600.0)
mi_addbhpnt("core",1.67,1800.0)
mi_addbhpnt("core",1.68,2000.0)
mi_addbhpnt("core",1.70,2200.0)
mi_addbhpnt("core",1.72,2400.0)
mi_addbhpnt("core",1.73,2600.0)
mi_addbhpnt("core",1.74,2800.0)
mi_addbhpnt("core",1.75,3000.0)
mi_addbhpnt("core",1.8091,10000)

-- primary winding (into the plane) coilP
mi_addnode(x1,y2)           -- add a new node at x,y
mi_addnode(x2,y2)
mi_addnode(x2,y3)
mi_addnode(x1,y3)
mi_addsegment(x1,y2,x2,y2)  -- add a new line segment from x1,y1 to x2,y2
mi_addsegment(x2,y2,x2,y3)
mi_addsegment(x2,y3,x1,y3)
mi_addsegment(x1,y3,x1,y2)
mi_addblocklabel(x1+xi,y2+yi) -- add a new blocklabel at x,y
mi_selectlabel(x1+xi,y2+yi)  -- select the blocklabel closest to x,y
mi_setblockprop("coilP",0,ElemSize,"coilP",0,1,N1) -- region properties
mi_clearselected()          -- clear previous selections

-- primary winding (from the plane) coilP
mi_addnode(x3,y2)           -- add a new node at x,y
mi_addnode(x4,y2)
mi_addnode(x4,y3)
mi_addnode(x3,y3)
mi_addsegment(x3,y2,x4,y2)  -- add a new line segment from x1,y1 to x2,y2
mi_addsegment(x4,y2,x4,y3)
mi_addsegment(x4,y3,x3,y3)
mi_addsegment(x3,y3,x3,y2)
mi_addblocklabel(x3+xi,y2+yi) -- add a new blocklabel at x,y
mi_selectlabel(x3+xi,y2+yi)  -- select the blocklabel closest to x,y
mi_setblockprop("coilP",0,ElemSize,"coilP",0,1,-N1) -- region properties
mi_clearselected()          -- clear previous selections

-- iron core
mi_addnode(x2,y1)           -- add a new node at x,y
mi_addnode(x6,y1)
mi_addnode(x6,y4)
mi_addnode(x2,y4)
mi_addsegment(x2,y1,x6,y1)  -- add a new line segment from x1,y1 to x2,y2
mi_addsegment(x6,y1,x6,y4)
mi_addsegment(x6,y4,x2,y4)
mi_addsegment(x2,y4,x2,y1)
mi_addblocklabel(x2+xi,y1+yi) -- add a new blocklabel at x,y
mi_selectlabel(x2+xi,y1+yi)  -- select the blocklabel closest to x,y
mi_setblockprop("core",0,ElemSize,"none",0,1,0) -- region properties
mi_clearselected()          -- clear previous selections

```

```

-- primary winding (into the plane) coils
mi_addnode(x4,y2)          -- add a new node at x,y
mi_addnode(x5,y2)
mi_addnode(x5,y3)
mi_addnode(x4,y3)
mi_addsegment(x4,y2,x5,y2) -- add a new line segment from x1,y1 to x2,y2
mi_addsegment(x5,y2,x5,y3)
mi_addsegment(x5,y3,x4,y3)
mi_addsegment(x4,y3,x4,y2)
mi_addblocklabel(x4+xi,y2+yi) -- add a new blocklabel at x,y
mi_selectlabel(x4+xi,y2+yi) -- select the blocklabel closest to x,y
mi_setblockprop("coils",0,ElemSize,"coils",0,1,N2) -- region properties
mi_clearselected()        -- clear previous selections

-- primary winding (from the plane) coils
mi_addnode(x6,y2)          -- add a new node at x,y
mi_addnode(x7,y2)
mi_addnode(x7,y3)
mi_addnode(x6,y3)
mi_addsegment(x6,y2,x7,y2) -- add a new line segment from x1,y1 to x2,y2
mi_addsegment(x7,y2,x7,y3)
mi_addsegment(x7,y3,x6,y3)
mi_addsegment(x6,y3,x6,y2)
mi_addblocklabel(x6+xi,y2+yi) -- add a new blocklabel at x,y
mi_selectlabel(x6+xi,y2+yi) -- select the blocklabel closest to x,y
mi_setblockprop("coils",0,ElemSize,"coils",0,1,-N2) -- region properties
mi_clearselected()        -- clear previous selections

-- background air-region
mi_addnode(x0,y0)          -- add a new node at x,y
mi_addnode(x8,y0)
mi_addnode(x8,y5)
mi_addnode(x0,y5)
mi_addsegment(x0,y0,x8,y0) -- add a new line segment from x1,y1 to x2,y2
mi_addsegment(x8,y0,x8,y5)
mi_addsegment(x8,y5,x0,y5)
mi_addsegment(x0,y5,x0,y0)
mi_addblocklabel(x0+xi,y0+yi) -- add a new blocklabel at x,y
mi_selectlabel(x0+xi,y0+yi) -- select the blocklabel closest to x,y
mi_setblockprop("air",0,ElemSize,"none",0,1,0) -- region properties
mi_clearselected()        -- clear previous selections

-- specify boundary conditions for the bounding air-region
mi_selectsegment(0.5*(x0+x8),y0) -- select the line closest to x,y
mi_selectsegment(x8,0.5*(y0+y5))
mi_selectsegment(0.5*(x0+x8),y5)
mi_selectsegment(x0,0.5*(y0+y5))
mi_setsegmentprop("tang",0,0) -- set the line segment properties
mi_clearselected()        -- clear previous selections

mi_zoomnatural()          -- view all geometry

mi_saveas("tmp_magn_ini.fem") -- save the file
mi_createmesh()           -- create a mesh
mi_analyse()              -- solve the problem
mi_loadsolution()         -- load the solution

-- post-processing
-- iterative computation of magnetizing current density
weight=-5;                -- weight of iteration
nriter=10;                -- max number of iterations
err=1; hh=0;              -- initial conditions
while err==1 do           -- iterative loop
  hh=hh+1;
  if hh==1 then jc2=Jcm; else jc2=jc1; end
  -- calculate the linking flux with core

```



### 4.3 Loaded transformer

By loading a transformer, the load current causes an opposing field to magnetizing field. Due to that the magnetic flux and also the induced voltage decreases. At the same time the seeming reduction is covered by an increased primary current, which in turn re-establishes the magnetizing field and sets the induced voltage up to the same level. The transformer operation point can be estimated according to the electrical equivalent circuit. The elements can be obtained from the FE model of a transformer at each operating frequency:

- Primary inductance,  $L_1 = \int_{S_p} A_z J dS \Big|_{I_1=1A, I_2=0A}$  (4.3)

- Secondary inductance,  $L_2 = \int_{S_s} A_z J dS \Big|_{I_1=0A, I_2=1A}$  (4.4)

- Mutual inductance,  $L_1 + L_2 + 2M = \int_{S_p+S_s} A_z J dS \Big|_{I_1=1A, I_2=1A}$  (4.5)

In the previous expressions,  $A_z$  stands for the magnetic vector potential and  $J$  for the current density. The surface integral is taken over an excited primary winding, an excited secondary winding or, respectively, both of them.

The important limit of the electromagnetic loading is the temperature limit and hot-spot temperature estimation, which will be discussed in the next section.

---

Q.12 How is related magnetic saturation, leakage flux and core losses to the electrical loading of the transformer?

Q.13 How is related magnetic saturation, leakage flux and core losses to the magnitude and frequency change of the primary voltage?

---

### 4.4 Temperature rise

The model setup of the problem on heat transfer is relatively inexperienced (Figure 4.1). First of all the 2D model can consider *heat dissipation* all the surfaces that are perpendicular to the cross-section of the electromagnetic device. Nevertheless, only three of these surfaces are considered as a subject of heat dissipation. Instead of using heat transfer coefficient, which can take describe heat dissipation by convection, the surface temperature is defined. This kind of boundary condition disregards the actual cooling condition it rather states that one way or another i.e. normal or forced cooling, the surface temperature has to be kept to the defined value.

Due to *geometrical simplification* the main insulation or the bobbin is ignored. Even though the simplification is acceptable in the computation of the problem on electromagnetism, the non-attendance has significant importance in the thermal computation. In the present case it can be assumed that the main insulation and the winding have the same equivalent heat transfer coefficient and the larger loss area can consider the heat transport from the end turns. It is considered that winding has much better heat transfer coefficient in the direction of current flow and worse in any other directions.

From thermal design point of view:

- The space for electric circuits is well utilized if they operate at the same loss density,
- Transformer's efficiency is largest when the copper losses equal to core losses,

- The thermal conductivity of a winding is a few orders lower than for laminated core, thus the winding cross-section determines the loading of the transformer.

The lua script for the heat transfer problem *PTC\_heat\_1.lua* is quite similar to the command lines, which handle the problem set up for electromagnetism. The new script consists also of a transformer geometry setup, a material characterization, a FE model setup, which includes boundary conditions and source definitions, solver and result analysis. The post-processing routine calculates iteratively the maximum current density that corresponds to the maximum temperature in the coil. The hot-spot temperature is considered to be on the line that divides the coil cross-section into two halves. The stability and converging speed is determined by *weight* variable.

```
-- solving a heat problem with Mirage 1.0

x0=0; y0=0; -- origin and the rest of the coordinates
x1=x0+aw; x2=x1+ww; x3=x2+cw; x4=x3+ww;
x5=x4+ww; x6=x5+cw; x7=x6+ww; x8=x7+aw;
y1=y0+ah; y2=y1+cw; y3=y2+wh; y4=y3+cw; y5=y4+ah;
xi=1e-4; yi=1e-4; -- displacement increment

-- additional parameters
ElemSize=0.001; -- max finite element size

-- parameter calculation
Aw = ww*wh; -- winding cross section
pcu1=(Jcm*1e+6)^2*rcu*Kf; -- copper loss density in the primary winding
pcu2=pcu1; -- copper loss density in the secondary
winding

-- create a new pre-processor for a thermal problem
newdocument()

-- problem definition
hi_probdef("meters","planar",1e-8,cl)

-- boundary definitions
hi_addboundprop("conv",2,0,0,20,10,0) -- convection
hi_addboundprop("Tamb",0,Tcoil-DeltaT,0,0,0,0) -- cooling surface temperature
hi_addboundprop("insulation",1,0,0,0,0,0) -- zero thermal flux

-- material definitions
hi_addmaterial("core",labmda_fe,labmda_fe,pfe)
hi_addmaterial("ins",0.12,0.12,0)
hi_addmaterial("air",labmda_air,labmda_air,0)
hi_addmaterial("coilP",labmda_eqv,labmda_eqv,pcu1)
hi_addmaterial("coils",labmda_eqv,labmda_eqv,pcu2)

-- primary winding (into the plane) coilP
hi_addnode(x1,y2) -- add a new node at x,y
hi_addnode(x2,y2)
hi_addnode(x2,y3)
hi_addnode(x1,y3)
hi_addsegment(x1,y2,x2,y2) -- add a new line segment from x1,y1 to x2,y2
hi_addsegment(x2,y2,x2,y3)
hi_addsegment(x2,y3,x1,y3)
hi_addsegment(x1,y3,x1,y2)
hi_addblocklabel(x1+xi,y2+yi) -- add a new blocklabel at x,y
hi_selectlabel(x1+xi,y2+yi) -- select the blocklabel closest to x,y
hi_setblockprop("coilP",0,ElemSize,1) -- define the region properties
hi_clearselected() -- clear previous selections

-- primary winding (from the plane) coilP
hi_addnode(x3,y2) -- add a new node at x,y
hi_addnode(x4,y2)
```

```

hi_addnode(x4,y3)
hi_addnode(x3,y3)
hi_addsegment(x3,y2,x4,y2) -- add a new line segment from x1,y1 to x2,y2
hi_addsegment(x4,y2,x4,y3)
hi_addsegment(x4,y3,x3,y3)
hi_addsegment(x3,y3,x3,y2)
hi_addblocklabel(x3+xi,y2+yi) -- add a new blocklabel at x,y
hi_selectlabel(x3+xi,y2+yi) -- select the blocklabel closest to x,y
hi_setblockprop("coilP",0,ElemSize,1) -- define the region properties
hi_clearselected() -- clear previous selections

-- iron core
hi_addnode(x2,y1) -- add a new node at x,y
hi_addnode(x6,y1)
hi_addnode(x6,y4)
hi_addnode(x2,y4)
hi_addsegment(x2,y1,x6,y1) -- add a new line segment from x1,y1 to x2,y2
hi_addsegment(x6,y1,x6,y4)
hi_addsegment(x6,y4,x2,y4)
hi_addsegment(x2,y4,x2,y1)
hi_addblocklabel(x2+xi,y1+yi) -- add a new blocklabel at x,y
hi_selectlabel(x2+xi,y1+yi) -- select the blocklabel closest to x,y
hi_setblockprop("core",0,ElemSize,1) -- define the region properties
hi_clearselected() -- clear previous selections

-- primary winding (into the plane) coils
hi_addnode(x4,y2) -- add a new node at x,y
hi_addnode(x5,y2)
hi_addnode(x5,y3)
hi_addnode(x4,y3)
hi_addsegment(x4,y2,x5,y2) -- add a new line segment from x1,y1 to x2,y2
hi_addsegment(x5,y2,x5,y3)
hi_addsegment(x5,y3,x4,y3)
hi_addsegment(x4,y3,x4,y2)
hi_addblocklabel(x4+xi,y2+yi) -- add a new blocklabel at x,y
hi_selectlabel(x4+xi,y2+yi) -- select the blocklabel closest to x,y
hi_setblockprop("coils",0,ElemSize,1) -- define the region properties
hi_clearselected() -- clear previous selections

-- primary winding (from the plane) coils
hi_addnode(x6,y2) -- add a new node at x,y
hi_addnode(x7,y2)
hi_addnode(x7,y3)
hi_addnode(x6,y3)
hi_addsegment(x6,y2,x7,y2) -- add a new line segment from x1,y1 to x2,y2
hi_addsegment(x7,y2,x7,y3)
hi_addsegment(x7,y3,x6,y3)
hi_addsegment(x6,y3,x6,y2)
hi_addblocklabel(x6+xi,y2+yi) -- add a new blocklabel at x,y
hi_selectlabel(x6+xi,y2+yi) -- select the blocklabel closest to x,y
hi_setblockprop("coils",0,ElemSize,1) -- define the region properties
hi_clearselected() -- clear previous selections

-- background air-region
hi_addnode(x0,y0) -- add a new node at x,y
hi_addnode(x8,y0)
hi_addnode(x8,y5)
hi_addnode(x0,y5)
hi_addsegment(x0,y0,x8,y0) -- add a new line segment from x1,y1 to x2,y2
hi_addsegment(x8,y0,x8,y5)
hi_addsegment(x8,y5,x0,y5)
hi_addsegment(x0,y5,x0,y0)
hi_addblocklabel(x0+xi,y0+yi) -- add a new blocklabel at x,y
hi_selectlabel(x0+xi,y0+yi) -- select the blocklabel closest to x,y
hi_setblockprop("air",0,ElemSize,1) -- define the region properties
hi_clearselected() -- clear previous selections

-- specify boundary conditions for the bounding air-region

```

```

hi_selectsegment(0.5*(x1+x7),y4) -- select the line closest to x,y
hi_selectsegment(x7,0.5*(y2+y3))
hi_selectsegment(x1,0.5*(y2+y3))
hi_setsegmentprop("Tamb",0,0) -- set the line segment properties
hi_clearselected() -- clear previous selections

hi_zoomnatural() -- view all geometry

Tmin=Tcoil-DeltaT
Tmax=Tcoil

hi_saveas("tmp_heat_ini.feh") -- save the file
hi_createmesh() -- create a mesh
hi_analyse() -- solve the problem
hi_loadsolution() -- load the solution

-- iterative computation of max current density
weight=0.025; -- weight of iteration
nriter=10; -- max number of iterations
err=1; hh=0; -- initial conditions

h=openfile("tmp_heat_res.txt","a"); -- open an output file for writing

while err==1 do -- iterative loop
  hh=hh+1; Tmax=0;
  if hh==1 then jc2=Jcm; else jc2=jc1; end
  ho_seteditmode("point")
  for kk = 0,10,1 do -- find maximum temperature in the line
    xp=x3+(x4-x3)/10*kk;
    T,Fx,Fy,Gx,Gy,kx,ky= ho_getpointvalues(xp,0.5*(y2+y3));
    if T>Tmax then Tmax=T; xpmax=xp; end
  end

  -- give a new value to current density
  if Tcoil-Tmax < -40 then jc1=jc2*0.5;
  elseif Tcoil-Tmax > 40 then jc1=jc2*2;
  else
  jc1=jc2+weight*(Tcoil-Tmax);
  end

  ho_seteditmode("area")
  ho_selectblock(0.5*(x3+x4),0.5*(y2+y3));
  Tave=ho_blockintegral(0); -- find an average temperature
  ho_clearblock()
  ho_close()
  rcu=rcu0*(1+0.004*(Tave-20));
  pcu=rcu*(jc1*1e+6)^2*Kf; -- give a new value for specific loss
  hi_modifymaterial("coilP",3,pcu)
  hi_modifymaterial("coils",3,pcu)
  hi_saveas("tmp_heat_ini.feh")
  hi_analyse() -- analyse updated problem
  hi_loadsolution()
  ho_showdensityplot(1,1,0,Tmax,Tmin) -- show the temperature plot
  Terr=abs(Tmax-Tcoil);
  -- conditions to finish the iterative loop
  if Terr<0.01 then err=0; end
  if hh>nriter then err=0; end
  -- messagebox(jc1)
end
for kk = 0,10,1 do -- find maximum temperature in the middle line
  xp=x3+(x4-x3)/10*kk;
  T,Fx,Fy,Gx,Gy,kx,ky= ho_getpointvalues(xp,0.5*(y2+y3));
  if T>Tmax then Tmax=T; xpmax=xp; end
end
ho_selectblock(0.5*(x3+x4),0.5*(y2+y3));
Tave=ho_blockintegral(0); -- find an average temperature
Aw1 =ho_blockintegral(1); -- find coil cross-section area
ho_clearblock()

```

```

write(h,xpmax," ",Tmax," ",Tave," ",pcu," ",jcl," ",Aw1,"\n")
closefile(h) -- close the file for writing
ho_savebitmap(filename) -- saves the current view to specified file

```

---

Q.14 Is the core losses temperature dependent?

Q.15 How suppose to be distributed coils that they could have equal thermal loading?

---

## 4.5 Computation sequence

The main task in the parameterised command routine is to carry out a series of computation and obtain numeric values into the table below that investigates the optimal proportions of the transformer. The numeric values from column 2 to 6 are directly obtained from FEMM and Mirage. The valued in column 7 is the product of columns 2 and 4 times copper fill-factor. The apparent power is estimated according to equation 3.18 and the losses are estimated from the loss density and the volume of the circuit.

Table 4.1 the result of the computation sequence of a transformer

Coil width	Coil cross-section area	Current density according to initial magnetization	Current density according to maximum temperature	Coil average temperature	Flux linkage for the primary winding	Ampere turns for the primary winding	Apparent power according to temperature rise	Copper losses	Core losses
ww	$Aw_1$	$J_{mag}$	$J_{max}$	$\vartheta_{coilave}$	$\Psi_1$	$NI_1$	$S_1$	$P_{cu}$	$P_{fe}$
mm	$mm^2$	A/mm <sup>2</sup>	A/mm <sup>2</sup>	°C	mVs	Aturns	VA	W	W
2	8.0	8.08	20.16	97.3	1.076	96.7	16.41	5.87	2.25
4	32.0	2.52	10.80	104.6	0.958	207.5	31.29	6.91	2.18
6	72.0	1.45	7.28	106.1	0.841	314.8	41.54	7.11	2.06
8	128.0	0.94	5.45	106.3	0.721	419.0	47.39	7.09	1.91
10	200.0	0.70	4.33	106.6	0.600	520.4	49.05	7.00	1.70
12	288.0	0.54	3.58	107.2	0.480	619.1	46.68	6.89	1.45
14	392.0	0.46	3.01	108.0	0.360	709.5	40.12	6.66	1.15
16	512.0	0.38	2.55	109.4	0.240	785.1	29.60	6.28	0.81
18	648.0	0.37	2.11	112.7	0.120	822.8	15.50	5.50	0.43

## 5 A three-phase transformer

This chapter gives a short overview of three-phase transformers, their construction and connections [6][26].

### 5.1 Construction

Similar to the constructions of a single-phase transformer, the three-phase transformer with a common core can be a shell-type (Figure 5.1), where the core surrounds the coil or a core-type (Figure 5.2), where the coil surrounds the core. The three-phase transformer with a common core is smaller than a bank of three single-phase transformers of the same total rating. This is due to symmetric loading of magnetic fluxes the magnetic core. One of the simplest core arrangements is to use a pair of E shell laminations to form a three phase core, which has also a drawback of poorly utilised 'slot' areas in the sides. Independent of a staked lamination or a wound core the three phase winding is always mounted and connected symmetrically.

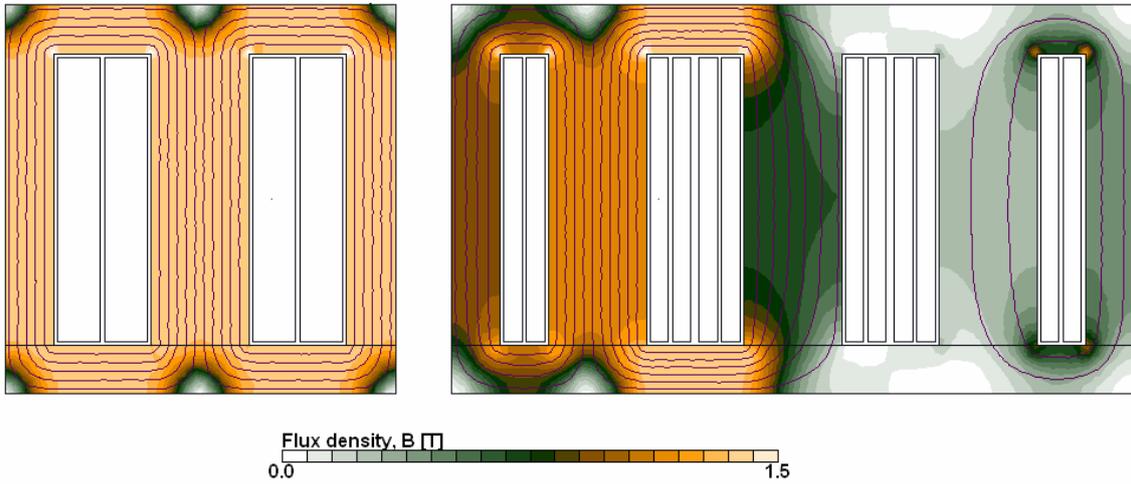


Figure 5.1 A shell type of a single-phase transformer and a three-phase transformer.

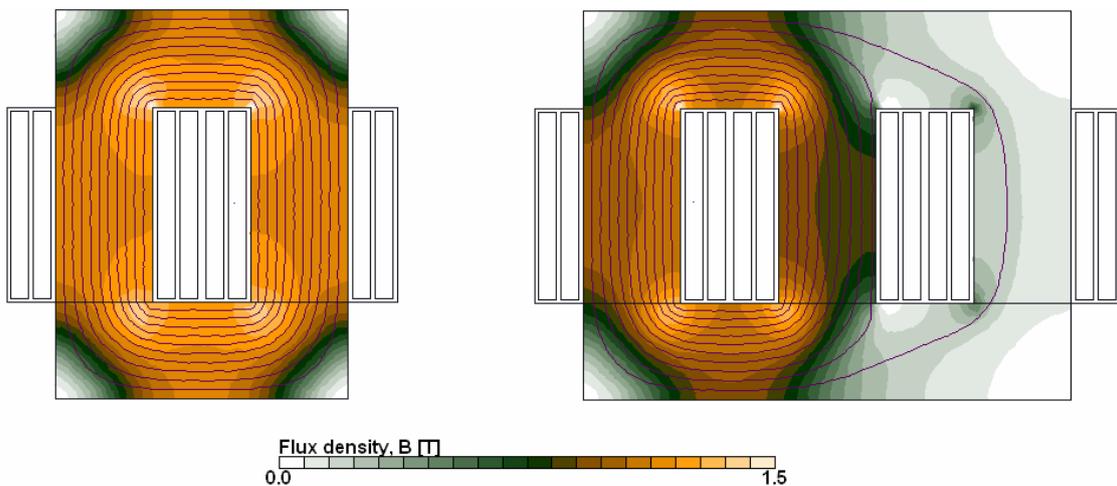


Figure 5.2 A core type of a single-phase transformer and a three-phase transformer.

The example calculation of the single-phase and a corresponding three-phase transformer show the magnetic flux path and flux density when the magnetizing current (in phase with flux) is maximum at phase-a and has the corresponding current density about  $0.2 \text{ A/mm}^2$ . The magnetic core is made of electromagnetic steel M530-50A and has the same width and height. The length of the core is made proportionally longer for the three phase counterpart. The slot area of a single phase in the three-phase transformer is an half of the available area in the corresponding single-phase transformer. At the same time the neighbouring phases contribute the magnetizing flux in the transformer. This example makes the single-phase transformer comparable with the three-phase transformer and visualizes the proportional relation as well as the geometry as the magnetization.

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Q.16 How should be transformer constructed in order to handle an unbalanced loading?

Q.17 How circulating currents influence transformer's performance?

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## 5.2 Electric circuits

The windings of the three phase transformer can be connected either to star or wye (Y) connection, delta ( $\Delta$ ) connection, or open star or delta connection. Figure 5.3 shows the schematic of the three-phase transformer where the primary is delta connected to the supply and the secondary is star (wye) connected to the load.

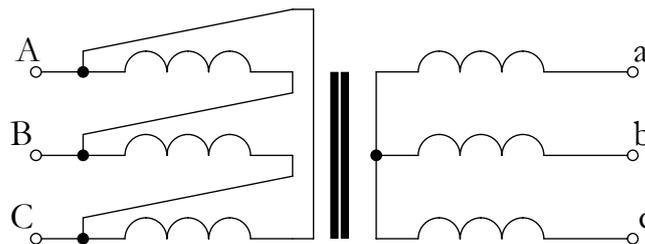


Figure 5.3 a schematic of a delta/ye transformer

- Y/Y connection enables the primary and secondary voltages to be in phase or inverted i.e. having  $180^\circ$  a phase shift. The neutral connections are optional, and their inclusion or omission gives different characteristics for the transformer. Sensitive to voltage unbalance.
- Y/ $\Delta$  connection causes always  $30^\circ$  phase shift that can be either leading or lagging between the secondary line-to-line voltages to the nearest primary line-to-line voltage. The neutral connections are optional, and their inclusion or omission gives different characteristics for the transformer. Less dependent on voltage unbalance and can operate efficiently with asymmetric supply. Possible connection is also open wye/open delta.
- $\Delta$ /Y connection causes  $30^\circ$  phase shift and has similar characteristics like the previous connection.
- $\Delta/\Delta$  connection causes always  $30^\circ$  phase shift that can be either leading or lagging between the secondary line-to-line voltages to the nearest primary line-to-line voltage. The neutral connections are optional, and their inclusion or omission gives different characteristics for the transformer. Possible connection is also open delta/open delta.

## 6 Experimental work

Prototyping transformer originates a great deal of practical questions that have been usually neglected or not exactly considered in the design process. The questions like mechanical tolerances margins, inter-winding and interlayer insulation, material and assembling properties. The design procedure can take into consideration of the manufacturing procedure and not only the tables of standard lamination shapes and the cross-sections of an enamelled wire. To understand the terminology is as important as to understand the technology.

### 6.1 Prototyping

Typically, when constructing a transformer the core is selected according to the application requirements, which is the outcome of the design process. This is followed by specifying the layout of the windings where the primary turns and the secondary turns are computed. The winding together with insulation is to be fitted into core window. The winding is usually wound on the bobbin and the stack of the lamination placed into the coil window. If everything fits together with respect to the tolerances then it is in safe hands to complete the prototype of a transformer.

### 6.2 Measurements

The transformer characteristics can be obtained from the equivalent circuit parameters. The parameters of the approximate equivalent circuit are readily obtained from open-circuit and short-circuit tests.

- The resistance of the winding(s) can be obtained with Ohmmeter.
- The primary inductance can be measured from open-circuited transformer. The inductance is a consequence of the iron and air in the magnetic field path, and is non-linear, which means that the inductance has different values under different conditions.
- Leakage inductance is inductance that results from the parts of the primary's magnetic field that does not link the secondary. This is an inductance from which the secondary can never draw energy, and represents a loss of effectiveness in the transformer. This can be obtained from the short-circuited transformer.
- The core losses can be found from the open-circuited transformer measurements by measuring the voltage across the winding  $U_0$ , the magnetizing current  $I_0$  and the input active power  $P_0$ .

$$R_c = \frac{U_0^2}{P_0} \quad (6.1)$$

$$Y_0 = \frac{I_0}{U_0} \quad (6.2)$$

$$X_m = \frac{1}{\sqrt{Y_0^2 - 1/R_c^2}} \quad (6.3)$$

- By a similar experiment the leakage impedance can be found from the short-circuited transformer. As the magnetizing impedance differs two orders of magnitude from the leakage impedance, the parallel coupled magnetizing circuit can be ignored.

$$R_e = \frac{P_{ss}}{I_{ss}^2} \quad (6.4)$$

$$Z_e = \frac{U_{ss}}{I_{ss}} \quad (6.5)$$

$$X_e = \sqrt{Z_e^2 - R_e^2} \quad (6.6)$$

These previously described measurements base on the simplified equivalent circuit that is shown in Figure 6.1. In order to keep the test voltage and current lower, the open-circuit test can be carried out from the secondary side and the short-circuit test from the primary side.

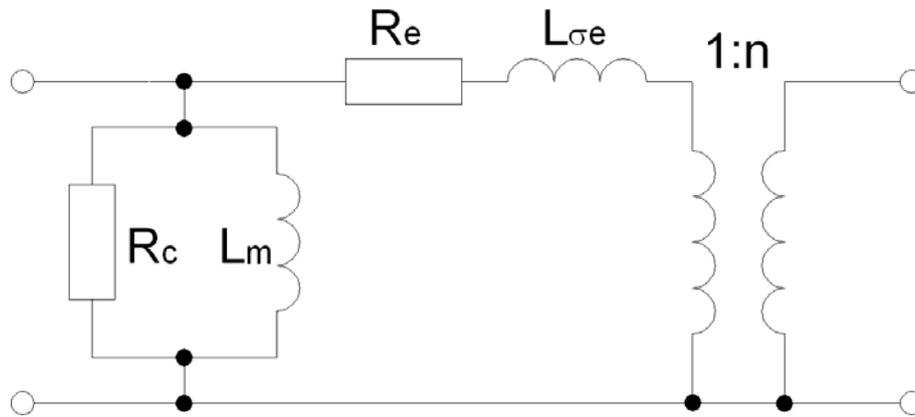


Figure 6.1 Simplified equivalent circuit

- The thermal loading can be estimated by winding resistance or thermocouple measurements.
- The efficiency is determined from the measurement of losses.

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