2. ELECTRO-MECHANICAL CONVERSION

2.1. General principles

Michael Faraday founded the principle of electro-mechanical energy conversion in 1831, when he studied the phenomena of electromagnetic induction. Electromagnetic induction is known as the generation of electromotive force (emf) in an electric conductor. The electromotive force can be generated in three different ways. First, the conductor is moving in the standstill magnetic field. Second, the magnetic field (of permanent magnets or electrically excited magnets) is moving toward the standstill conductor. Third, the conductor and magnets are at standstill, but the excitation current of magnets is varying. The direction of the induced electromotive force can be determined by the rule of the right hand (Fig. 2.1).

When the lines of the magnetic field are directed to the hollow of the hand and the thumb shows the direction of the conductor’s movement, the fingers are directed toward the induced electromotive force.

![Figure 2.1. Determination of the direction of the induced electromotive force (a) and the influence of force on the conductor (b)](image)

A higher density of the magnetic flux and a higher velocity of the conductor movement will cause a higher electromotive force in the conductor.

\[ E = B l v \sin \alpha , \]

where \( E \) is the induced electromotive force in volts (V), \( B \) is the density of the magnetic flux or induction in tesla (T), \( l \) is the longitude of the moving conductor in meters (m), \( v \) is the velocity of movement (m/s) and \( \alpha \) is the angle between the direction of movement and flux lines of the magnetic field. When the conductor is moving toward the flux lines of the field, the angle \( \alpha = 0 \) and the induced electromotive force will be equal to zero. The induced electromotive force has a maximal value, when the direction of movement is perpendicular to the flux lines of the magnetic field. If the velocity of movement is considered the derivative of the position’s time variation \( v = ds/dt \), the formula of the induced electromotive force can be expressed as follows.
The product of the magnetic flux density $B$ and the area $ls$ is the magnetic flux. Therefore the formula for the electromotive force can be expressed by help of the time derivative of the magnetic flux.

$$E = B l \frac{ds}{dt} \sin \alpha .$$  \hspace{1cm} (2.2)

Consequently, the generated electromotive force is proportional to the variation of the velocity of the magnetic flux.

When the electric conductor with the generated electromotive force forms a closed loop, the electric current $i$ in this loop will increase. The value of the current can be determined by the law of Ohm $i = E/R$, where $i$ is the current in amperes (A) and $R$ is the electric resistance in ohms ($\Omega$). The direction of the current can be determined by the rule of H. F. E. Lenz.

**The influence (direction) of the current generated by the induced electromotive force is regularly directed against the phenomenon that caused the current in the closed loop (e.g., against the movement of the electric conductor or changing of the magnetic field).**

Consequently, the electric conductor in a magnetic field is working as a force generator. The direction of the generated force can be determined by the rule of the left hand (Fig. 2.1).

**When the lines of the magnetic field are directed to the hollow of the hand and the fingers are directed toward the induced current, the thumb shows the direction of the generated force.**

A higher density of the magnetic flux and bigger longitude of the conductor will result in a greater force imposed on the conductor.

$$F = B i l \sin \alpha \hspace{1cm} (2.4)$$

The force could be directed against the movement (against the direction of velocity) or could be in the same direction as the velocity. First, when the external force moves the conductor, the electromotive force is induced in this conductor and the current carrying closed-loop of an electrical circuit could be formed. In this case, the principle of an electrical generator is realized and the mechanical input energy will be transformed to the electrical output energy.

$$P_{meh} = F v = B i l v \sin \alpha = E i = i^2 R = P_{el} \hspace{1cm} (2.5)$$

Secondly, when in the conductor situated in the magnetic field, the current $i$ is generated by an external source of the electromotive force, then the force on the conductor and as a result, mechanical movement will be generated. In this case, the principle of an electrical motor is realized and the electrical input energy will be transformed to the mechanical output energy.

An electric machine is a reversible energy converter. The same electric machine can work as an electric motor or a generator of electrical energy. Electric motors are used in drives of technological machines (fans, pumps, compressors etc.) or in drives of different technological equipment. Modern technological machines need electric motors with flexible speed and torque control and high efficiency. But the function of an electric motor is not the only function of electric machines. The optimal motion control of technological machines is a very urgent problem, to save energy in industrial processing. The kinetic or potential energy of moving masses could be converted into electrical energy also by electric machines. For example, cranes could generate electrical energy during the lowering of a load. In the same
way, electrical energy will be generated during the braking of vehicles with electrical traction drives.

Modern electrical drives convert flexibly electrical energy to mechanical and vice versa mechanical to electrical. To realize this main function of electrical drives, different types of power converters are needed. In electrical drives, converters are used to switch over the voltage polarity and regulate the value of voltage, current and frequency. Converters can be different sine wave or pulse modulated energy sources. Today these converters and regulators will be realized in the best way by help of power semiconductor devices.

Depending on their current source, electrical machines are classified as DC and AC machines. AC machines are classified as synchronous and asynchronous (or induction) machines. Electrical and magnetic circuits are two basic components of all electric machines. To achieve a higher efficiency of a machine, the magnetic circuit must have a higher magnetic conductivity (lower reluctance) and the electrical circuit a higher electrical conductivity (lower resistance). Therefore magnetic circuits are made from ferromagnetic materials and windings of machines are made from copper. To suppress the eddy currents, magnetic circuits of AC machines are made from thin insulated sheets of steel. The magnetic circuit of DC machines consists of massive steel and permanent magnets. An important component of a magnetic circuit is the air gap between the stator and the rotor of a machine. For higher efficiency, the air gap must be minimized.

Different losses appearing during the energy conversion process in an electric machine are as follows:

- **copper losses** in the windings of an electric machine
- **magnetic core losses** due to hysteresis and eddy currents in the magnetic core
- **ventilation losses** due to aero-dynamical resistance
- **bearing losses** due to friction in bearings.

### 2.2. Direct current electric machine

The excitation winding or permanent magnets on the poles of a machine generate the magnetic field in the DC machine (Fig. 2.2). The poles (N, S) are fixed to the yoke of the machine. The yoke is a part of the magnetic core and housing of the machine. The part of the machine, where the magnetic field is generated, is called an inductor. The current-carrying conductors of the armature are moving in this magnetic field. Continuous rotating of the armature is possible when the direction of the current is commutated according to the position of the conductors in relation to the pole. This process is realized by a commutator that consists of copper contacts and sliding carbon brushes.
In conventional DC machines a mechanical commutator is used. But in modern electrical machines, the commutator can be realized by use of semiconductor switches. The part of a machine that consists of the commutator and a current carrying winding is called an armature. Ordinarily, the inductor called a stator is at standstill and the rotating armature is called the rotor of the machine. During the commutation process, the direction of the armature current is alternating. Therefore the current in the DC machine windings is, in fact, the alternating current.

An equivalent scheme of the loaded DC machine (Fig. 2.3) will be used to describe the working principle of a DC machine.

The constant excitation flux is the assumption for the following description.

\[
\begin{align*}
U_a &= i_a \cdot R_a + L_a \frac{d i_a}{dt} + E; \\
E &= k\Phi \cdot \omega; \\
J \frac{d\omega}{dt} &= T_m - T_s; \\
T_m &= k \cdot \Phi \cdot i_a; \\
T_s &= f(\omega); \\
\omega &= \frac{d\Phi}{dt},
\end{align*}
\]

where \( U \) – armature voltage, \( i \) – armature current, \( R \) – armature resistance, \( L \) – armature inductivity, \( E \) – back electromotive force, \( k \) – machine constant, \( \Phi \) – magnetic flux of
excitation winding or permanent magnets, $J$ – equivalent moment of inertia, 
$\omega$ – rotating speed, $T_m$ – torque of machine, $T_s$ – loading torque, $\varphi$ – angle of shaft rotation.

According to the equations, the armature current is the function of supply voltage and back electromotive force. In a linear model, the electromotive force is proportional to the rotation speed of the machine and the torque of the machine is proportional to the armature current. In real electric machines, the magnetic core saturation phenomena also exist and nonlinear functions must be considered. The variables of a DC machine are mutually connected by differential equations.

The system of differential equations must be solved to analyze the operation of a DC machine. Today, numerical calculations are most commonly used. It is well known that the simplest method for solving differential equations is the Euler method. Its use is illustrated on the basis of a DC machine.

For example, we can assume that during a short time interval $\Delta t$, the variation of motor’s back electromotive force is insufficient, to influence the motor armature current. If the time interval $\Delta t$ is very short, then the increment of the armature current is much smaller than the value of the armature current ($\Delta i << i$) and can be calculated from the equation:

$$\Delta i = \frac{U - k \Phi \cdot \omega_i - i \cdot R}{L} \cdot \Delta t,$$

(2.7)

where $\omega_i$ – rotation speed before the time interval $\Delta t_i$. After the time interval $\Delta t_i$, the current $i_i$ grows by $\Delta i_i$ and the new value of the armature current can be calculated from the equation:

$$i_{i+1} = i_i + \Delta i_i.$$  

In the same way, the rotation speed $\omega_i$ and increment $\Delta \omega_i$ can be calculated, with an assumption that current $i_i$, and consequently, the torque variations during the time interval $\Delta t_i$ are small.

$$\Delta \omega_i = \frac{k \Phi_i - T_s(\omega_i)}{J} \Delta t_i,$$

(2.8)

where $k \Phi_i = T_m$ – torque of machine and $T_s(\omega_i)$ – load torque before the time interval $\Delta t_i$. After the time interval $\Delta t_i$, the rotation speed $\omega_i$ grows by the speed increment $\Delta \omega_i$.

$$\omega_{i+1} = \omega_i + \Delta \omega_i.$$  

(2.9)

The increment of the rotation angle

$$\Delta \varphi_i = \omega_i \cdot \Delta t_i$$

(2.10)

and the rotation angle after the time interval $\Delta t_i$

$$\varphi_{i+1} = \varphi_i + \Delta \varphi_i.$$  

(2.11)

It can be concluded that for the assumed small value of $\Delta t$, the differential equations of DC machine can be numerically solved by help of a step-by-step algorithm not only in the case of the described linear model but also for different nonlinear models.

2.3. Alternating current induction machine
The induction machine of the alternating current can work as an electric motor or generator. The induction motor is the most widely used electrical machine in the world. A motor consists of a standstill stator and a rotating rotor. The stator is intended for the generation of rotating magnetic field. The rotation speed of the rotor depends on the rotating speed of the magnetic field, which is the function of the pole number and the frequency of the supply voltage. The rotation speed of the magnetic field is called the synchronous speed $\omega_0$ (rad/s) of the machine.

$$\omega_0 = \frac{2\pi f}{p} = \frac{2\pi n_0}{60}$$  \hspace{1cm} (2.12)

where $\omega_0$ – synchronous speed (rad/s), $f$ – frequency of supply current, $p$ – number of poles and $n_0$ – synchronous speed in rpm (rotations per minute).

The magnetic field of an AC induction motor is shown in Fig. 2.4. In the case of the 2-pole machine (Fig. 2.4, a) and the supply current frequency 50 Hz, the rotation speed of the magnetic field is 3000 rpm or 314 rad/s. With the 4-pole machine (Fig. 2.4, b), the rotation speed of the magnetic field is 1500 rpm or 157 rad/s. With the 6-pole machine, the rotation speed of the magnetic field is 1000 rpm or 105 rad/s.

To operate the induction motor, the rotating magnetic field of a stator must exist, but also the current in rotor winding must be generated. In the case of the induction motor, the rotor current is induced by the rotating field of the stator on the principle of electromagnetic induction. The English language term of the induction motor is based on the above. The stator and rotor windings of the induction motor are mutually electromagnetically bounded. According to the principle of electromagnetic induction, the electromotive force is generated if the magnetic field around the conductor is varying. Therefore the rotating speed of the magnetic field of the stator and the speed of rotor must be different to induce the electromotive force and current in the rotor winding. This difference is called the slip of the rotor and can be denoted by $s$. The rotor slip is calculated as the relative difference of synchronous $\omega_0$ and rotor speed $\omega$.

$$s = \frac{\omega_0 - \omega}{\omega_0} = \frac{n_0 - n}{n_0}$$  \hspace{1cm} (2.13)

The induction motor is called also an asynchronous motor because of its rotor speed differences from the synchronous speed of magnetic flux. The slip of a standard AC induction motor is a few per cent. The induction motors of lower power have higher values of slip.
The steady state model of the AC induction motor is based on the 1-phase equivalent scheme of the machine. Different equivalent schemes can be used, but the T-form equivalent scheme (Fig. 2.5) is most popular. The stator and rotor currents calculated from this equivalent scheme can be used to calculate the power and torque on the shaft converted by the machine. The parameters of the induction motor shown on the equivalent scheme are as follows: \( R_s \) – active resistance of the stator winding, \( L_s \) – leakage inductance of the stator winding, \( L_m \) – the main inductance of the machine, \( L_r \) – leakage inductance of the rotor winding, \( R_r \) – active resistance of the rotor winding.

The following Kirchhoff equations for the stator and rotor circuits of the induction machine will be completed on the basis of an equivalent scheme in Fig. 2.5.

\[
U_1 = I_s \cdot R_s + j \omega \left( I_s \cdot L_s + I_m \cdot L_m \right),
\]
\[
0 = I_r \cdot \frac{R_r}{S} + j \omega \left( I_r \cdot L_r + I_m \cdot L_m \right). \tag{2.14}
\]

The rotor current is calculated as the function of the electromotive force \( E \) and equivalent scheme parameters.
Calculated from the equivalent scheme, the active power of the rotor circuit is proportional to the power of the rotor active resistance \( R_r/s \). The real power of the rotor circuit is equal to the dissipated power of the active resistance \( R_r \). The mechanical power of the induction motor will be calculated as the power difference

\[
P_{\text{mech}} = ml^2_r \left( \frac{R_r}{s} - R_r \right) = ml^2_r R_r \left( \frac{1-s}{s} \right),
\]

where \( m \) is the phase number (conventionally \( m = 3 \)).

The assumed angular speed of the rotor is \( \omega_s \), and therefore the torque on the motor shaft can be calculated by the formula:

\[
T = \frac{P_{\text{mech}}}{\omega} = ml^2_r R_r \left( \frac{1-s}{s} \right) / \omega.
\]

The angular speed of the rotor \( \omega \) is the function of slip \( s \)

\[
\omega = \omega_s (1 - s) = 2\pi \cdot f (1 - s) / p,
\]

where \( \omega_s \) is the synchronous speed of the rotor and \( p \) the number of poles. The torque of the induction motor can be calculated by a simple formula

\[
T = \frac{p m}{2\pi f} l^2_r \cdot \frac{R_r}{s}.
\]

The speed can be calculated from the following differential equation:

\[
J \frac{d\omega}{dt} = T - T_s,
\]

where \( J \) is the equivalent moment of inertia and \( T_s \) is loading torque. Note that the kinetic energy of the equivalent moment of inertia is equal to the summary kinetic energy of all moving parts driven by the electric machine.

The mechanical characteristic of an electric machine (the steady state function of speed and torque) can be calculated from equations 2.14 to 2.19 (see 2.6). Note that these calculations are based on the simple 1-phase equivalent scheme and on average values of machine variables. Therefore it is impossible to describe the rotation of the magnetic field, alternating current in windings and other dynamic effects in a machine. The other disadvantage of this equivalent scheme is the very approximate description of rotor circuits. The skin effect in a rotor winding is typical of the squirrel cage induction motors and must be considered when the mechanical characteristics are studied. The simplified equivalent scheme in Fig. 2.5 can be used if the induction motor has a wound rotor or if the slip variation is very small.

The nature of the skin effect in a squirrel cage rotor winding is as follows. The skin effect will appear in an electric conductor with a large cross-section area of the conductor and a relatively high frequency of current. Due to the skin effect, the current cross-sectional distribution will be heterogeneous and the higher current density will be in the external layers of the conductor. A higher frequency will lead to a rise in current concentration on the external layer of the conductor. The frequency in the rotor winding rods depends on the rotor slip. During the process of motor start, the slip varies from \( s = 1 \ldots s_n \), where the nominal slip \( s_n \approx 0.05 \). The rotor frequency is equal to the supply frequency, if \( s = 1 \) and becomes smaller...
with the rotor slip. As the result of the skin effect, the equivalent cross-section area of the conductor is reduced and its equivalent active resistance increases. The active resistance of a rotor is an important parameter for an induction motor and every change of its value will influence the form of its mechanical characteristics.

### 2.4. Synchronous and step-machines

The stator windings of the synchronous motor generate the rotating magnetic field in the same way as the stator windings of the induction motor (Fig. 2.6). However, in the synchronous motor the magnetic field is generated in the rotor by permanent magnets or by the excitation winding. These two magnetic fluxes clutch each other (stator N poles clutch with rotor S poles and vice versa), resulting in the rotor rotation with the rotation of the stator magnetic flux. The rotor speed is equal to the speed of field rotation, i.e. to the synchronous speed. The excitation current the excitation winding on the rotor must be conducted to the rotor via slip rings on the rotor shaft. With permanent magnets, slip rings are not needed.

Depending on the rotor design, synchronous machines are classified as salient pole and hidden pole machines. In the first case, the poles are clearly observable, the air gap has the minimum value above the poles and the maximum value between the poles. In the second case, the rotor surface is smooth and air gap has an equal value around the rotor. The salient pole machine can also work without any excitation, because it has different reluctance values (due to difference in the air gap) around the rotor. The poles are clutching because of the minimum energy potential in the position of the minimum reluctance. The machine is called a reluctance machine.

![Salient poles synchronous machine](image)

*Figure 2.6. General principles and design of a synchronous machine*

The synchronous machine generates the torque only in the synchronous operation mode. Therefore the direct-on-line start of the synchronous motor is not possible. To start the synchronous motor, the separate start winding of induction current or the smooth frequency controlled start process can be used.

**Step motors** have the same operation principle as synchronous motors, but sequential pulses supplied from a special pulse source rather than by 3-phase sinus wave voltages, generate the rotating magnetic field. The low frequency of supply pulses could be the reason of non-continuous rotation of the machine. Step motors are used in relatively low power servo drives for the positioning of different mechanisms. The advantage of step motors is the fixed function between the rotor position and the number of supply pulses on the windings, because
the additional position sensor is not needed. To achieve a higher accuracy of positioning, step motors with a higher number of poles are used. The disadvantage of step motors is their relatively low efficiency. Therefore the power range of step motors is under 1 kW. The design of step motors depends on the excitation principles used. The permanent magnet excitation, salient poles with variable reluctance and combined excitation principles are used.

2.5. Three-phase rotating magnetic field

The rotating magnetic field of AC machines is generated by the three-phase stator windings system. Three windings are equally distributed around the stator’s inner surface, supplied from the three-phase source with sine wave voltages. The voltages of different phases are shifted in time scale by 1/3 (or 120°) of the period (see Fig. 2.7). The spatially shifted windings with time-shifted sine wave currents generate the moving (rotating) magnetic field. The moving or rotating magnetic field can be generated also by other numbers of phases. For low power electric machines, the two-phase windings and 90° shifted supply voltages are used. Principally, the five-phase windings and supply voltages can be used.

The three-phase supply voltage system is most common for electricity networks. Accordingly, the three-phase electric machines are most common for electric drives. These machines can be supplied directly from a supply network or from power converters, e.g. from frequency converters. Direct on-line switching of high power induction motors will cause a current shock and voltage drop in supply lines. To avoid this, several types of starting circuits (e.g. star-delta circuit) or soft-starters with power semiconductor switches are used. The current in delta-connected windings at the same voltage is $3\times$ times higher than for a star connection. A proper connection of windings and the right value of supply voltage are important to guarantee motor operation.

Note that alternating voltages and currents can be determined by different variables. The instantaneous voltage, the magnitude and phase of periodical sine voltage, average value or root mean square (rms) value of alternating voltage can be determined. The rms value characterizes the working capacity of current or voltage (i.e. power $P = i^2R$ or $P = U^2/R$). For sine voltage, the average and rms values are the following functions of the sine wave magnitude.

$$U_{av} = 2 \frac{U_{max}}{\pi} = 0.63 U_{max}$$
$$U_{rms} = \frac{U_{max}}{\sqrt{2}} = 0.71 U_{max} \quad (2.21)$$

The relation of the rms value and average value is called the form factor. With the sine wave, the form factor is $K_f = 1.11$.

![Diagram of three-phase rotating magnetic field](image_url)
The electrical 1-phase energy supply based on the line L1, neutral N and protection earth PE conductors. The 3-phase supply system consists of three line conductors L1, L2, L3, one neutral N and one protection earth PE conductor (Fig. 2.8). The neutral conductor is the current carrying conductor in the 1-phase system, but not always in the three-phase system. Sometimes the neutral conductor can be connected with the central point of the star connected load to guarantee the symmetrical voltages on the load. The PE conductor must be connected to the body of the electrical device to guarantee human safety.

Figure 2.8. Star and delta connections of AC machine stator windings
2.6. Speed and torque control of electric machines

For the user, the mechanical characteristics (the speed-torque functions) of the motor and the load are the most important features of the electric drive. The speed-torque diagram for a DC machine can be calculated from equations 2.6. The speed-torque diagram of a load machine depends on the features of the machine. For the hoist machines, the torque is independent of the speed, for fans, pumps and compressors, the torque is proportional to the square of the speed, etc. (Fig 2.9, a). Operation point of the drive is determined as the crossing point of speed-torque diagrams of the motor and load machine. The varying of armature voltage can control the speed of a drive, but in many cases, to achieve the high values of speed or torque, the excitation current of the motor must also be varied (Fig. 2.9, b).

According to the classical design, DC motors can be classified as motors with parallel or series excitation windings or with combined compound windings. Motors with parallel excitation windings are used in drives, which are mainly loaded with constant torque. Commonly, their speed must be independent of the torque value. DC motors with serial excitation winding are used in drives that are mainly loaded with constant power. The excitation winding is connected serially with the DC motor armature winding and carries the same current. Consequently, a higher loading torque and a higher motor current will automatically lead to a higher excitation field and as a result, a higher torque of the motor. When the loading torque diminishes, the speed increases and the motor torque is reduced. Such mechanical features are most suitable for traction drives. Therefore the serially excited DC motors are used traditionally in the drives of trams, trolleybuses, electric cars, trains etc. Today’s advancement in power electronics and microprocessor engineering offers us many new possibilities for flexible and optimal regulation of drive speed and torque in conformity with the needs of every type of a working machine. Modern electronics and control devices help us to form such speed-torque diagrams of a drive that we really need. Therefore special motor designs (e.g., the serial excitation winding) are not so important any more.

![Figure 2.9. Speed-torque diagrams of a DC motor at different load curves and excitation voltages](image)

Figure 2.9. Speed-torque diagrams of a DC motor at different load curves and excitation voltages
Historically, to solve the problem of the speed control of AC motors was very complicated, the solutions appeared of low quality, expensive or of low efficiency. In fact, with varying frequency, the reactance of a winding will also be varying. The stator current must be limited, because the saturation effect in a ferromagnetic material will cause very high losses in windings and a very fast rise in temperature. Therefore in addition to the frequency control, the voltage or current control is needed. Without suitable semiconductor switches and microprocessor control devices, the frequency control is too complex and expensive. Today the high efficiency frequency control is the main method for smooth speed control of AC motors. By help of the microprocessor-controlled devices and software-based control algorithms it is possible to optimize the efficiency of a motor at different values of speed and torque.

The speed of an AC induction motor can be controlled by different methods of frequency control. Speed control based on the reference function of voltage $U = f(f)$ is the most common. The simplest way is to regard that $U/f = \text{constant}$ and the function $U = f(f)$ is linear. Fig. 2.10, a shows the mechanical characteristics, i.e. the speed-torque diagrams, of AC an induction motor. Diagrams are calculated for different supply frequencies and for the regulation rule $U/f = \text{constant}$. At higher speed values, the maximum torque is approximately constant, but in the range of low frequencies and speeds, the torque becomes essentially smaller and the rotor slip becomes much higher. The operation points on the speed-torque surface are shown for the constant torque ($T = \text{const}$) as well for the linear ($T = k\omega$) and square speed function torque ($T = k\omega^2$). Consequently, the control rule $U/f = \text{const}$ is useful if the load of a motor becomes smaller with a decrease in the speed. For different load characteristics, the control rule $U = f(f)$ must be varied. For the constant torque, the load characteristic of the motor maximum torque must be also approximately constant. This can be achieved by the corresponding control of the motor voltage.

![Figure 2.10. Speed-torque diagrams of AC machines](image-url)

Figure 2.10. Speed-torque diagrams of AC machines a) speed control of AC induction motor by help of linear function ($U/f = \text{const}$); b) speed control of synchronous motor
The optimal voltage control for the frequency-controlled AC induction motor is essential to achieve the needed load capacity and high efficiency of the drive. Due to the limited value of the maximum torque, the dynamic load peaks can stop (stall) the rotor. To avoid this, the voltage of stator must have a sufficient value and could be controlled as the function of the torque. Note that the generated torque of an induction motor is proportional to the square of the stator voltage. On the other hand, too high values of voltage can cause saturation in the magnetic circuit of a machine and consequently higher energy losses and a faster temperature rise.

In modern frequency converters, different rules of voltage control can be selected. Fig. 2.11 shows the pattern diagrams of possible \( U = f (f) \) control rules. The uncompensated linear or nonlinear as well the shifted linear or nonlinear compensation functions can be used. To compensate the torque drop in the low speed range, an initial compensation voltage \( (U_0) \) must be applied. Nonlinear compensation is appropriate for the square load speed-torque diagram \( (T_k = k\omega^2) \), e.g., for the speed control of fans, pumps, compressors etc. A frequency converter user can select settings that suit for the drive used. Settings can be pre-selected before the drive operation or automatically adjusted during the operation.

![Figure 2.11. Patterns of voltage-frequency control functions](image)

The speed of a synchronous motor is also controlled by means of a frequency converter. The speed is exactly proportional to the frequency of the supply current. The speed-torque diagrams are direct horizontal lines, which means that the speed value is independent of the motor torque (Fig. 2.10, b). The maximum torque of a synchronous motor is limited by the value of the motor torque capacity. The supply voltage must also be regulated as the function of frequency. The dynamic load peaks can stop (stall) the rotor if the voltage is too low.

Note that by help of modern control devices, the AC synchronous motor can easily replace the DC motor, with the same technical features, including the same speed-torque diagrams. If the stator windings are not commutated with the constant line frequency, but as a function of rotor position, the machine will have the same operation principle as a DC motor. The only difference is that instead of the mechanical commutator, a semiconductor switch-based converter is used and the rotor position is detected with a special sensor. The motors of that type are called **electronically commutated DC motors** or **brush-less DC motors**. These motors have a wide use in industry: in servo drives of machine tools and industrial robots.
2.7. Dynamical models of electric machines

The dynamical model of a DC machine. As a rule, in the calculations, a DC machine is considered as a machine, the poles of which have infinity longitude. In this case, the current direction in the armature winding is independent of the position of the armature. In fact, the armature is moving from one pole to the other and the direction of the magnetic flux is alternating. Therefore, the direction of the current must also be changed to keep the same rotation direction. It means that traditional models of a DC machine do not describe the real dynamics of a machine, i.e. the alternating current flows in the armature windings of the DC machine. An ordinary model does not enable us to explain why the motor is continually rotating.

Figure 2.12 shows the rotor comprising only one loop winding and a two-segment commutator on a two-pole motor. The mutual action of the excitation flux $\Phi$ and the current carrying armature winding generates the force vector $F$ on the armature. The tangential component $F_m$ of the force vector causes the rotating torque $T_m = F_m r$, where $F_m$ is the function of rotation angle $F_m = F \cos \alpha$. If $\alpha = 90^\circ$, the force $F_m = 0$. After that the sign and direction of the force will change. To continue the armature rotation, the current direction must also be reversed. In conventional DC machines, a rotor winding normally comprises a large number of series-connected loops known as winding sections. Each section is connected to two adjacent segments of the commutator. During the time the brushes short-circuit the two associated segments, the direction of the current in the latter is reversed. The section commutates.

![Figure 2.12. The dynamical model of the DC machine with the armature current commutator](image)

The dynamic process in the DC machine is described by the following system of equations (2.22):

$$
\begin{aligned}
\frac{du}{dt} &= \frac{1}{L}(u - iR) \\
\frac{di}{dt} &= (U_m - k_m \Phi \omega) \cdot \text{sgn}(\cos(\alpha + \alpha_h)) \\
\frac{d\omega}{dt} &= \frac{1}{J}(k_m \Phi i \cdot \cos \alpha) \\
\frac{d\alpha}{dt} &= \omega
\end{aligned}
$$

(2.22)
In equations 2.22, \( U_m \) is the motor supply voltage (DC voltage), \( u \) – instantaneous voltage (AC voltage) on the winding section, \( \alpha \) – angle of rotation, \( \alpha_0 \) – shift angle of brushes relative to the neutral line of the machine, \( k_m \) – motor design coefficient (that considers the number of poles and winding sections, etc.), \( R \) – armature active resistance, \( L \) – armature inductance, \( J \) – armature and load machine summary moment of inertia, \( \omega \) – angular rotating speed and \( \Phi \) – excitation flux. The transients of the armature winding current and rotation speed calculated from equations 2.22 are shown in Fig. 2.13.

The dynamical model of the AC induction motor is very complicated. Descriptions of this model are given in special literature concerning modern electrical drives. Note that the dynamical model of the AC induction motor is the basis to realize the vector control principle of the induction motor. Figure 2.14 shows the AC motor stator current and rotor speed transients on starting the motor. In general, the current and speed transients of different motor types are very similar because the main principle of all electrical machines is the same.

The starting process of low power electrical machines is a relatively short-time process, from tens of millisecond to a few seconds. Therefore, a much higher starting current is allowed than the nominal current of machine \( I_{\text{start}} = 3...6 \ I_{\text{nom}} \). The starting current of high power electrical machines must be limited, because the current shock causes essential voltage drops in a supply line, disturbing normal functioning of other electrical equipment.
2.8. Brake and generator modes of operation

Different braking methods can be used to stop an electric motor. For the reverse braking of an AC induction motor, two supply lines of the stator winding must be interchanged. The direction of field rotation changes and the slip value rises up to 200% ($s = 2$). The torque sign of the motor becomes opposite; the motor will brake on rotation masses and the speed is reduced. The reverse braking causes a high current shock ($I_{\text{brake}} > 6 I_{\text{nom}}$). The other problem is motor stoppage, because braking is very fast and after the zero-speed, the start to the opposite direction will follow. Therefore special zero-speed sensors and automatic switch-off must be used.

The most common braking method of electric motors is the dynamical or resistive braking. For AC induction motors (as the standard), the AC supply voltage must be switched off and the stator windings must be switched to the DC source. In this situation the rotor is rotating, but the magnetic flux is at standstill. The current generated in the moving rotor winding and the standstill magnetic flux results the generation of the braking torque. The dynamic braking of an AC induction motor is called DC braking because of the direct current in the stator winding. The speed-torque diagrams and different braking modes of an AC induction motor are shown in Fig. 2.15.

![Figure 2.15. Patterns of mechanical characteristics of an AC induction machine for motor operation, brake and generation modes](image-url)
Note that for the high-speed range, the dynamical braking torque has a relatively small value. The braking torque has a maximum value in the low-speed range close to the zero-speed. Therefore the dynamical braking is used after the motor switch-off and coast to stop by freewheeling after the speed drops under 0.2-0.5 of the nominal speed. In a special case of dynamical breaking, the stator windings are non-symmetrically short-circuited or switched via semiconductor diodes. The term dynamical braking is used because in this braking mode, the motor generates torque only if rotating. At standstill, the torque is equal to zero.

If the speed of the rotor exceeds the speed of field rotation (synchronous speed), when the motor is driven with the working machine (e.g. during the crane load lowering), the motor will operate as a generator. This operation mode is called recuperative braking (or regenerative braking). Torque direction is opposite to torque direction in the motor operation mode and the energy will be returned to the motor supply source.

With a DC motor in the dynamical braking mode, armature windings are switched off from supply voltage and closed on the resistor. The electromotive force generated in windings causes the braking current and braking torque. During the DC motor at standstill, the torque is equal to zero. The most useful braking mode is recuperative braking because energy can be returned to the supply source.

A new trend in the energy conversion is to use ultra-capacitors for energy storage. Then the braking energy can be stored during drive braking and reused during the next start of the drive.