4. SYNTHESIS of AUTOMATIC CONTROL SYSTEMS

As synthesis of the automatic control systems, the design of a system proceeded from pre-determined requirements is meant. The synthesis is more complicated process than it is the analysis of an existing system, therefore that the synthesis does not have a unique solution, so as to the same result different methods could lead - for example the structure of the system or the control algorithms could be varied. In one case, one has to deal with the solution on the energy level, in the second case on the information level, [2].

Here it is suitable to repeat the formulation of the control task – the objective of the control is to take the definition point of the system from initial point A to the point B, according to the control task; satisfying quality requirements simultaneously. An essential abstract here is, that the motion in the state space does not mean the geographical change of a mechanism’s parts, but the change of quantities describing entire state of the system.

An automatic control system is usually designed around a previously known control object, a motor, for instance. If the parameters of the object are known in advance, then in the design process only the regulator parameters are determined, or the regulator will be synthesised.

The synthesis of the regulator is required in three cases

- The system is structurally unstable
- the accuracy of the system is not sufficient
- the transient processes of the system do not scope with quality requirements

Earlier the trial-and-error method was used in the synthesis, which had set higher requests to the designers, however, the development of the computer technology enables preliminary simulation of systems, reducing the cost of the systems and requests to the designers. Simulation could be made by various software, MatCad, MATLAB, etc. for instance, That enables carry out the calculations as in the time so in frequency and in state spaces, but on the background of all of it one must not forget, that simulations could be inaccurate (look division 3.3.1.4.)
4.1. Selection of the suitable regulator.

The following consideration holds for simple automatic control systems. Connecting together a stable control object and stable control device might not give a stable system. The same time while connection of two unstable devices might give a stable system. From this, the complexity of the synthesis could be understood, wherefore in the practice certain system configuration are widespread, which provide structural stability in case of negative feedback [3].

Table 4.1. Structurally stable systems

<table>
<thead>
<tr>
<th>Obj.</th>
<th>P</th>
<th>PT₁</th>
<th>PT₂</th>
<th>I</th>
<th>IT₁</th>
<th>P</th>
<th>PT₁</th>
<th>I</th>
<th>P</th>
<th>PT₁</th>
<th>P</th>
<th>P</th>
<th>PT₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reg.</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>PT₁</td>
<td>I</td>
<td>P</td>
<td>PT₁</td>
<td>I</td>
<td>IT₁</td>
<td>PI</td>
</tr>
</tbody>
</table>

In the design process the regulator will be selected so, that the parameters of the regulator, lying in given limits, provide the stability of the system. Thereby the rule is taken into account, that the order of the regulator must not exceed the order of the control object. The reason of this rule could be explained as follows:

For each order there corresponds one time constant in the system, and the more time constants are in the system, the slower, and vibratory is the system. The system time constants are compensated by regulators, or they are attempted to be reduced out. In case if for the control of a second order system a third order regulator will be used, then one more time constant will be implemented into the system, in addition of two existing already, which were attempted to reduce, or - the regulation is not as efficient as it could be.

4.2. Comparison of different regulators

To investigate most widespread PID-family regulators like P, PI, and PID it is suitable to use a PT₃ control object, parameters of which were $K_1 = K_2 = K_3 = 1$, $T_1 = 5\text{ s}$, $T_2 = 1\text{ s}$, $T_3 = 0.2\text{ s}$. Such automatic control system is represented in the figure 4.1.
In the comparison of the regulators one has to follow, in the abovementioned reason, the action in the control input as by the variation of the control input as well of the variation of the disturbance, so as the response of the system to this signal is different. In the figure 4.2 the characteristics of the given above system are represented for different gains of P-regulator as for the control input as well for the disturbance.

If the existing PT\textsubscript{3}-block has been stable independently, but with a slow transient process, and then if updated with an P-regulator, the reaction of the process to the change of the control input has speeded up, but by the small gain of the regulator the system turned inaccurate. Increasing of the gain the process speeded up even more,
also the accuracy has improved, but the vibratory processes with over-regulation arose, which in some cases are undesired. With further increase of the regulator’s gain the stability margin will be exceeded and the output will oscillate with increasing amplitude, the mean of which presents the desired output value.

Considering the reaction of the system to a disturbance by given gains of regulators on might see, that the reaction speed does not depend on the regulators, however, the regulator is able to compensate the impact of the disturbance in certain limits, not being able to reduce this impact to zero. Compensation is the more efficient the grater is the regulator’s gain, but this will involve oscillating processes.

Comparing P-regulator with other regulators one can see, that the regulators with I-block are accurate or they reach the set value of the system in the output, but they are inevitably accompanied with oscillating process. The same happens by the response to the disturbance, where the regulators with I-block are able to compensate its impact fully. Comparing PI-regulators with PID regulators one has to mention as a difference the fact, that D-block provides faster response, although it is not clearly to see in examples given.

Causal actions of parts of a PID-regulator are explained in the figure 4.5. Provided that the control object (figure 4.4.) was previously operating in a settled mode without
 regulator’s help. Thereafter in one time moment a disturbance \( n(t) \) arose to the system, which was reflected in comparison block before the regulator, inducing declination in the regulators input, to which all three blocks reacted, summing up their actions. The red action shown in the figure is the response on the proportional block to the deviation, which is amplified depending on the gain of regulator. The integrating block starts to increase the output of the regulator smoothly (represented in blue), but the differentiating block reacts with large amplitude jump as the fastest (represented in green), that immediately reduces the effect of the disturbance. By the time when the differentiating block starts loosing its influence, the proportional and integrating blocks are so much “recovered”, that they take over the disturbance’s compensation. If at one time moment a full compensation of the disturbance is reached, the deviation in the regulators input disappears, on which the differentiation block reacts with a reverse jump, after disappearing of what the integrator remains holding the control action in output, with which the disturbance is compensated. Differentiating and proportional blocks remain not applied, so as the disturbance activating them is missing, however, the integrator remains summing-up the virtual deviation.

As a result, those actions the regulators of PID-family are widespread. A family are they called therefore, that in most cases as well as P-regulators, so PI, PD and ID regulators are realised with PID-regulators, in which the not needed blocks will be deactivated.
4.3. Adjustment of a PID-regulator

For the adjustment of a regulator different possibilities exist, from which one, the analytical, was considered by determining of the stability domains. Analytical methods usually contain calculations in considerable amount and giving as result the answer about suitability of the stability domain or parameters configuration of certain system for certain task given with initial assignment. Therefore in the practice abstract methods are used, which give with less calulations a preliminary set of parameters, which will be optimised in the further process of simulation or fitting. For the comparison of different adjustments mutually the integral criteria are used. Below are some most widespread in practice for the regulator’ adjustment are considered.

4.3.1. Ziegler-Nichols’s method

Ziegler-Nichols's method is used for closed loops (in figure 4.4) proceeding from the stability margin or in other words – the gain of a P-regulator will be increased until the output of the system is oscillating permanently without damping or amplifying. The corresponding gain is called critical gain and the oscillation period -
critical period. Equations for calculations are given in the table 4.2. [6]

Table 4.2. Parameters of a regulator by Ziegler-Nichols.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>P-regulator</th>
<th>PI-regulator</th>
<th>PID-regulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_R$</td>
<td>$0.5 \cdot K_{krit}$</td>
<td>$0.45 \cdot K_{krit}$</td>
<td>$0.6 \cdot K_{krit}$</td>
</tr>
<tr>
<td>$T_I$</td>
<td>-</td>
<td>$0.83 \cdot T_{krit}$</td>
<td>$0.5 \cdot T_{krit}$</td>
</tr>
<tr>
<td>$T_D$</td>
<td>-</td>
<td>-</td>
<td>$0.125 \cdot T_{krit}$</td>
</tr>
</tbody>
</table>

4.3.2. CHR-method

CHR in an abbreviation from the authors names – Chien, Hrones and Reswick. CHR was developed from the Ziegler-Nichols's method for implementation of certain quality requirements of open systems. Using the aperiodic step response, the conditional parameters of the process will be determined [6]

- time lag of the process $T_H$;
- time constant of the process $T_0$;
- gain of the process $K_p$.

\[ T_H + T_D \]
The gain of the process is calculated by the formula

\[ K_p = \frac{\Delta y}{\Delta s}. \] (4.1)

Suitable for the regulator parameters are calculated depending on the control objective and the regulator’s type, therefore the equations are assembled in the table 4.3.

Table 4.3. Calculation of the regulator’s parameters by CHR-method

<table>
<thead>
<tr>
<th>Regulator</th>
<th>Parameter</th>
<th>Aperiodic process</th>
<th>with 20%-over-regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>For the control</td>
<td>For the disturbance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{T_0}{K_p T_H} )</td>
<td>( \frac{T_0}{K_p T_H} )</td>
</tr>
<tr>
<td>P</td>
<td>( K_R )</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>PI</td>
<td>( K_R )</td>
<td>0.35</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>( T_1 )</td>
<td>1.2 ( T_0 )</td>
<td>4 ( T_H )</td>
</tr>
<tr>
<td>PID</td>
<td>( K_R )</td>
<td>0.6</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>( T_1 )</td>
<td>( T_0 )</td>
<td>2.4 ( T_H )</td>
</tr>
<tr>
<td></td>
<td>( T_D )</td>
<td>0.5 ( T_H )</td>
<td>0.42 ( T_H )</td>
</tr>
<tr>
<td>PD</td>
<td>( K_R )</td>
<td>( \frac{T_0}{K_p T_H} )</td>
<td>1.8 ( \frac{T_0}{K_p T_H} )</td>
</tr>
<tr>
<td></td>
<td>( T_D )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 4.1.**

Let it be given a control object which is described by a PT\(_3\)-block and the process of which is desired to accelerate, allowing 20% over-regulation in the output thereby. The problem is realised by a PID-regulator For the investigation of difference between adjustment of the control and of the disturbance the ISE-criterion should be used- The control system under consider is presented by the automatic control system structure in figure 4.1 and in the figure 4.7, the step response of a corresponding PT\(_3\)-block is given, from which the following parameters could be determined:

\[ J_{ISE} = 380; \]

\[ K_p = 1; \]
\( T_H = 0.7 \text{s}; \)
\( T_0 = 7.9 \text{s}. \)

From the table 4.3. are selected corresponding for the performance specification formula and found on their basis regulator’s parameters are assembled in the table 4.4.

Table 4.4. Calculated parameters of the regulator

<table>
<thead>
<tr>
<th></th>
<th>( K_R )</th>
<th>( T_I, \text{s} )</th>
<th>( K_I, s^{-1} )</th>
<th>( T_D, \text{s} )</th>
<th>( K_D, s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juhtimisele</td>
<td>10.72</td>
<td>10.67</td>
<td>1.00</td>
<td>0.329</td>
<td>3.53</td>
</tr>
<tr>
<td>Häiringule</td>
<td>13.54</td>
<td>1.40</td>
<td>9.67</td>
<td>0.294</td>
<td>3.98</td>
</tr>
</tbody>
</table>

Simulation results are presented in figure 4.8. and 4.9. and in the table 4.5.

Figure 4.8. Transient characteristic of a control-optimised system a) by changing of the control signal, b) by changing of the disturbance
Figure 4.9. Transient characteristic of an system optimise to the disturbance compensation
a) by changing of the control signal, b) by changing of the disturbance

Table 4.5. Integral evaluations of the regulator’s adjustments

<table>
<thead>
<tr>
<th></th>
<th>Control optimised</th>
<th></th>
<th>Disturbance optimised</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>To the control</td>
<td>To the</td>
<td>To the control</td>
<td>To the</td>
</tr>
<tr>
<td></td>
<td></td>
<td>disturbance</td>
<td></td>
<td>disturbance</td>
</tr>
<tr>
<td>IAE</td>
<td>-1.1711</td>
<td>-0.9244</td>
<td>-0.9214</td>
<td>-0.6790</td>
</tr>
<tr>
<td>ISE</td>
<td>1.0134</td>
<td>0.0455</td>
<td>0.9679</td>
<td>0.0256</td>
</tr>
</tbody>
</table>

If to evaluate the adjustment of a regulator by integral criteria, one may notice that the susten which is disturbance-optimised reacts better to the change of the control signal than the control-optimised system, but if to compare the transients following the change of the control signal, then disturbance-optimised system takes more time for the stabilisation, despite of a less over-regulation. Hereby one has to look at the systems performance specification – is a longer regulation period acceptable? Also one has to consider, that empiric formula give preliminary parameters for the regulator adjustment, which are fitted to the corresponding system in the process of following after-fitting. Investigating the adjustment from the disturbance point of view, the as expected, the disturbance-optimised system behaves more efficient than the control optimised one, still the recommendation for systems after-optimisation remains valid

4.3.3. Determination of regulator’s parameters on the Bode diagram.

The presumption of the method is the existence of open system Bode diagram and for the adjustment of the regulator graphical solution method is used.[3]

By the lack of experience, one has to do with the trial-error method to meet the quality requirements of the transient process, but the adjustment of the system for the frequency processes proceeds better by this method than by two previous.

If to study the influence of regulator’s gain on the Bode diagram, one may notice, that the increase of the gain increases and decrease of it decreases the amplitude-frequency
characteristic, not changing thereby the phase-frequency characteristic of the system.

Alternatively, it follows from here, that it is possible to achieve with the proportional block of the regulator the required amplitude reserve.

The gain of a PI-regulator has similar properties like P-regulators one. Provided, that most of real objects are PT\textsubscript{n}-blocks, the time constant of an integrating block is approximated to the largest time constant of the system. As a result of it, one has good reaction to the change of the control signal.

PD-regulator is used when the control object shows integrating action. In this case, the rising effect of the phase-frequency characteristic are applied. As the integration time constant was approximated to the largest time constant of the system, the same will be made with differentiation time constant.

Using PID-regulator, the time constants will be adjusted to the largest time constant of the control object, holding the condition $T_i > T_d$ . Also it must be taken into account the pirate time constants of the real integrating and differentiating blocks, which are tried to be selected minimal possible to minimise the distortions.
4.3.4. Amplitudoptimum

Considering an abstract automatic control system on may assume, that an ideal response of the control signal was in case if the transfer function of the whole system \( W = 1 \). In this case, the output of the system will follow the input of the system exactly [9].

As process, one PT\(_n\)-block could be considered, by which most of the drives in use could be described and the transfer function of which is

\[
W_P(p) = \frac{1}{a_0 + a_1 \cdot p + a_2 \cdot p^2 + \ldots}.
\] (4.2)

Suitable regulator for this process is PID-regulator, transfer function of which could be represented in form

\[
W_R(p) = \frac{r_2 \cdot p^2 + r_1 \cdot p + r_0}{2 \cdot p}.
\] (4.3)

Bearing in mind the objective of the synthesis \( W = 1 \), the sets of equations are elaborated for the calculation of the parameters of the equation (4.3), which assembled in the table 4.6.

In case if the transfer function is given in the form

\[
W_P(p) = \frac{K_p}{\prod_i (1 + T_i \cdot p)},
\] (4.4)

then, by condition that \( T_1 \gg T_2 = \sum_{j=2}^{n} T_j \) this process could be describes as PT\(_2\)-block

\[
W_P(p) = \frac{K_p}{(1 + T_1 \cdot p)(1 + T_2 \cdot p)}.
\] (4.5)
or in situation, where \( T_1, T_2 \gg T_2 = \sum_{j=3}^{n} T_j \) is valid, the process could be represented as PT\(_3\)-block

\[
W_p(p) = \frac{K_p}{(1 + T_1 \cdot p)(1 + T_2 \cdot p)(1 + T_3 \cdot p)}.
\]

Synthesis algorithm, proceeding from this presentation is brought in table 4.6.

Table 4.6. Determination of the regulator’s parameters on the bases of amplitude optimum

<table>
<thead>
<tr>
<th>Process</th>
<th>Regulator</th>
<th>Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{r_1 \cdot p + r_0}{2 \cdot p} )</td>
<td>( r_0 = a_0 \cdot \frac{a_1^2 - a_0 \cdot a_2}{a_1 \cdot a_2 - a_0 \cdot a_3} )</td>
<td>( r_0 = a_0 \cdot \frac{a_1^2 - a_0 \cdot a_2}{a_1 \cdot a_2 - a_0 \cdot a_3} )</td>
</tr>
<tr>
<td>( \frac{r_1 \cdot p + r_0}{2 \cdot p} )</td>
<td>( r_1 = a_1 \cdot \frac{a_1^2 - a_0 \cdot a_2}{a_1 \cdot a_2 - a_0 \cdot a_3} - a_0 )</td>
<td>( r_1 = a_1 \cdot \frac{a_1^2 - a_0 \cdot a_2}{a_1 \cdot a_2 - a_0 \cdot a_3} - a_0 )</td>
</tr>
</tbody>
</table>
| \( \frac{r_2 \cdot p^2 + r_1 \cdot p + r_0}{2 \cdot p} \) | \( \begin{bmatrix}
\begin{array}{ccc}
a_0 & -a_0 & 0 \\
-a_1^2 + 2 \cdot a_0 \cdot a_2 & -a_2 & a_1 \\
a_2^2 + 2 \cdot a_0 \cdot a_4 - 2 \cdot a_1 \cdot a_3 & -a_4 & a_3 \\
\end{array}
\end{bmatrix} 
\) | \( \begin{bmatrix}
\begin{array}{ccc}
a_0 & -a_0 & 0 \\
-a_1^2 + 2 \cdot a_0 \cdot a_2 & -a_2 & a_1 \\
-a_3^2 + 2 \cdot a_0 \cdot a_4 - 2 \cdot a_1 \cdot a_3 & -a_4 & a_3 \\
\end{array}
\end{bmatrix} 
\) |
| \( \frac{K_p}{(1 + T_1 \cdot p)(1 + T_2 \cdot p)} \) | \( K_R \cdot \frac{1 + T_R \cdot p}{p} \) | \( K_R = \frac{1}{2 \cdot K_p \cdot T_2} \)
| \( T_R = T_1 \) |
By the adjustment of the amplitude optimum special cases may appear, two of which are presented in the table 4.7 [6].

Table 4.7. Special cases of the adjustment of amplitude optimum [6]

<table>
<thead>
<tr>
<th></th>
<th>Tüüp A</th>
<th>Tüüp B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer function of the open system</td>
<td>$K_R \cdot K_P \cdot K_I \cdot \frac{1}{p \cdot (1 + p \cdot T_1)}$</td>
<td>$K_R \cdot K_P \cdot \frac{1}{(1 + p \cdot T_1) \cdot (1 + p \cdot T_2)}$</td>
</tr>
<tr>
<td>Dependence between damping rate and gain</td>
<td>$K_R \cdot K_P \cdot K_I = \frac{1}{4 \cdot \chi^2 \cdot T_1}$</td>
<td>$K_R \cdot K_P = \frac{(T_1 + T_2)^2}{4 \cdot D^2 \cdot T_1 \cdot T_2} - 1$</td>
</tr>
<tr>
<td>Adjustment to the damping $\chi = 0.707$</td>
<td>$K_{\text{opt}} = \frac{1}{2 \cdot K_P \cdot K_I \cdot T_1}$</td>
<td>$K_{\text{opt}} = \frac{(T_1 + T_2)^2}{2 \cdot K_P \cdot T_1 \cdot T_2} - \frac{1}{K_P}$</td>
</tr>
<tr>
<td>Special cases</td>
<td>$W_0 = \frac{K_R \cdot K_P}{p \cdot T_1 \cdot (1 + p \cdot T_1)}$</td>
<td>$T_2 \gg T_1$</td>
</tr>
<tr>
<td></td>
<td>$K_{\text{opt}} = \frac{T_1}{2 \cdot K_P \cdot T_1}$</td>
<td>$K_{\text{opt}} = \frac{T_2}{2 \cdot K_P \cdot T_1}$</td>
</tr>
</tbody>
</table>

Parameters, found by these algorithms are not the parameters, which could be adjusted on the regulator. They should be previously be modified.

Example 4.2.
In the synthesis process of a PID-regulator the transfer function of the regulator was found as

\[ W_R(p) = \frac{r_2 \cdot p^2 + r_1 \cdot p + r_0}{2 \cdot p} = \frac{r_2}{2} \cdot p^2 + \frac{r_1}{2} \cdot p + \frac{r_0}{2} \cdot p. \]

The transfer function of a real regulator is given in the form

\[ W_{RR}(p) = K_p + \frac{K_i}{p} + K_D \cdot p, \]

parameters of which could be found by the comparison of the transfer function coefficients.

\[ K_p = \frac{r_1}{2}, \quad K_i = \frac{r_0}{2}, \quad K_D = \frac{r_2}{2}. \]

### 4.4. Synthesis of MIMO systems

Devices and drives are widely spread in the industry, by which a number of output variables, which in turn could be internally dependent, are controlled simultaneously. Digital regulators are suitable for these systems, for the synthesis of which matrix and vector calculus are used, however, in some cases, cascade regulation, for instance, the synthesis could be made as in time space as well in the frequency space also.
Figure 4.16. MIMO-system of parallel structure

Figure 4.17. MIMO-system with cascade regulation of serial structure

Systems of parallel structure are synthesised in the time space more seldom than the systems of serial structure, which are especially spread for the drivers control, providing good dynamic to the drives and could be at the same time simply synthesised and realised. The advantages of this system come evident therefore, that several regulators are used, where the regulator of the auxiliary variable compensates one time constant, accelerating the dynamics of this part, whereas, at the same time, the main regulator must compensate the rest of the system. However, this is not the only advantage. The fact is that the regulator of auxiliary variable is faster than the main regulator, and therefore the disturbances of the input side are compensated by the auxiliary regulator without letting them to the output.

In the synthesis of cascade regulators the following stages are to be distinguished:

First, the regulator of the auxiliary variable will be selected, so that with it the desired transient characteristic in the inner part of the system will be reached. The quality requirements to the auxiliary variable could differ from requirements set to the output. In addition, it is possible to give up the fixed accuracy, so as the fast dynamics is most important requirement to the regulator.

The adjusted subsystem is considered as a one-piece block, therefore the selection of the main regulator proceeds similarly to the auxiliary regulators one.

Selection of these regulators is made by methods considered in section 4.3 or other equivalent methods. However, the synthesis of the systems of parallel structure differs
from the synthesis of the systems of serial structure, therefore, that different parallel structures are distinguished [6]:

forward or P-structure,
reversed or V-structure.

Figure 4.18. Parallel structures a) P-structure, b) V-structure

In such systems, $W_{11}$ and $W_{22}$ describe main loops, and $W_{12}$ and $W_{21}$ coupling loops.

For the control of systems regulators are used which represent several jointly acting regulators. They are called diagonal regulators and compensating regulators.

Diagonal regulators are used when the following conditions are met:

one has to do with a process of P-structure
the system has equal number of inputs and outputs;
the coupling loops are not integrating kind.
Diagonal matrix inherits its name from the presentation mode of the system regulator.

\[
W_R = \begin{bmatrix}
W_{R1} & 0 \\
0 & W_{R2}
\end{bmatrix}.
\] (4.6)

The synthesis of the diagonal matrix is easier than of the compensating regulator, containing the following stages:

- determination of main loops
- selection of regulators.
- assignment of the regulator’s parameters.

Under the determination of main loops the operation is meant, where it will be arranged which input entity imposes most directly which output variable, or considering figure 4.20, when it was arranged that \( s_1 \) imposes \( y_1 \) directly and \( s_2 \) imposes \( y_2 \), although it could be arranged opposite, – that \( s_1 \) imposes \( y_2 \). In this case, the locations of the transfer functions in the figure will change, system by itself do not change.

By the selection of regulators one precedes similarly to the synthesis of one-loop systems, just the determination of the parameters is made on the different basis. First, the static coupling factor of the process will be determined

\[
S_0 = \frac{W_{12}(0) \cdot W_{21}(0)}{W_{11}(0) \cdot W_{22}(0)}.
\] (4.7)

Following, the transfer function of the main loops are determined, conceiving, that another main loop does not exist

\[
W_1 = \frac{W_{R1} \cdot W_{11}}{1 + W_{R1} \cdot W_{11}},
\] (4.8)

\[
W_2 = \frac{W_{R2} \cdot W_{22}}{1 + W_{R2} \cdot W_{22}}.
\] (4.9)
For the determination of conventional transfer functions of the forward loops, if the main loops are influencing mutually, the following equations are used

\[ W_1^*(p) = W_{11}(p) \cdot [1 - S_0 \cdot W_2(0)], \]

(4.10)

\[ W_2^*(p) = W_{22}(p) \cdot [1 - S_0 \cdot W_1(0)]. \]

(4.11)

Now it is possible to represent the forward loop (chain?) in the following form:

![Equivalent circuit of the 1. main loop](image)

Figure 20. Equivalent circuit of the 1. main loop, if \( s_2 = 0 \)

It is easy to synthesise the regulator’s parameters for such system by the amplitude optimum.

In case of compensating regulators the auxiliary regulators are used, which compensate the impact of coupling loops and thereafter the main regulators will be adjusted like usual regulators. As a disadvantage of this regulator is that they are complicated to adjust, especially if one has to do with changing parameters [6].

![System with compensating regulator](image)

By the above-mentioned reasons, the synthesis of such systems is suitable for experts in automatic control and therefore it will not treated here further.
Exercises to the Chapter 4.

Exercise 10.

The control object is an IT₂-block with parameters $K = 1.25 \text{ s}^{-1}$, $T_1 = 0.2 \text{ s}$, $T_2 = 3.9 \text{ s}$. Select suitable for the object regulator and assign its parameters. Optimise the obtained results.

Exercise 11.

The control object is a PT₃-block with parameters $K = 4.23$, $T_1 = 0.12 \text{ s}$, $T_2 = 0.78 \text{ s}$, $T_3 = 9.78 \text{ s}$. Select suitable for the object regulator and synthesize its parameters by the following methods:

- Ziegler-Nichols's method;
- Chien, Hrones ja Reswick's method;
- Amplitude optimum method;

Which difference could be noticed in the step responses with different sets of parameters?

Exercise 12.

The control object is an IT₁-block with parameters $K = 1 \text{ s}^{-1}$, $T_1 = 0.78 \text{ s}$. Select suitable for the object regulator and synthesize its parameters by amplitude optimum method.

Exercise 13.

A DC motor is desired to be used in speed-controlled drive. The parameters of the drive are:

- resistance of the motor $R = 2.23 \Omega$;
- reactance of the motor $L = 0.214 \text{ H}$;
- motor factor (?) $c = 0.011 \text{ Vs}$;
- moment of inertia of the motor $J = 2.4 \cdot 10^{-3} \text{ kgm}^2$. 
In the calculations the effects of the load and electro-motor force could be omitted, also the distortions induced by the time lag of the power converter. Such system is represented in the figure 4.17. For the control of current use the P-regulator and also for the speed P-regulator. Synthesise parameters of these regulators.